# TRANSIENT MATHEMATICAL MODEL OF BOTH SIDE SINGLE PASS PHOTOVOLTAIC THERMAL AIR COLLECTOR

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## ABSTRACT

A mathematical model and solution procedure of a single pass photovoltaic thermal air collector (PVT) with Compound Parabolic Concentrator (CPC) and fins with both sides of the absorber for predicting the thermal and combined photovoltaic thermal performance of the system is presented. The air which is the working fluid flows between top glass and absorber plate and between absorber and bottom plates. The mathematical model is composed of five couple unsteady nonlinear partial equations which are solved by using Gear implicit numerical scheme. The temperatures of the circulated air as a function of distance in the flow direction for both sides are predicted. Results at solar irradiance of 400 W/m$^2$ show that the combined pv/t efficiency is increasing from 26.6 % to 39.13 % at mass flow rates varies from 0.0316 to 0.09 kg/s.

**Keywords:** photovoltaic, thermal, air, collector, transient, model, performance, implicit numerical scheme.

## INTRODUCTION

Solar energy is one of the most important source of renewable energy that world needs. The major applications of solar energy can be classified into two categories, Thermal system (T) and Photovoltaic system (PV) cell. Normally, these systems are used separately. In conventional solar thermal system, external electrical energy is required to circulate the working fluid through the system. In conventional photovoltaic system, high incident solar radiation on (PV) panel should give high electrical output. However, high incident will increase the temperature of the solar cells and that will decrease the efficiency of the panel. Therefore, to achieve both higher cell efficiency and higher electrical output we must cool the cells by removing the heat in some way. To eliminate an external electrical source from the thermal system and to cool the cells in photovoltaic system we integrate a photovoltaic panel with solar air / water heater collector, this can make when photovoltaic cells pasted directly on the flat plate absorber. This type of system is called photovoltaic-thermal collector (PV/T) or hybrid (PV/T) and this system has advantage such as it can be used to generate both thermal and electrical energy simultaneously, cooling PV improves efficiency, heat can be used in space heating or for drying system, it is less costly than two separate unites and it is very attractive in case the available roof surface is limited.

A number of theoretical as well as experimental studies have been made on (PV/T) systems with air and liquid as working fluid. Kern and Russell [1] are the first who give main concept of PV/T collector using water or air as the heat removal fluid. Hendrie and Raghuraman [2] have made a comparative experimental study on photovoltaic thermal collectors with liquid and air as working fluid. Raghuraman [3] presented numerical methods predicting the performance of liquid and air PV/T collector. Sopian *et al.* [4] have successfully demonstrated the improved performance of a steady state double pass collector over the single pass collector due to efficient cooling of the photovoltaic cell. Garg and Adhikari [5] reported the performance analysis of hybrid photovoltaic-thermal collector with integrated compound parabolic concentrator (CPC) troughs; this case indicated that the total efficiency with CPC is higher than system without CPC. Zondag *et al.* [6] compared the efficiency of seven different design types of photovoltaic thermal collectors. Mohd.Y.Hj.Othman *et al.* [7] design a new model of double pass PV/T air collector with CPC and fins and they studied its performance over range of operating conditions.

## MATHEMATICAL MODEL

The solar collector considered in this paper is shown in Figure-1. It has three essential static components: a glazing on the top, a plate containing numerous solar cells and a bottom plate.

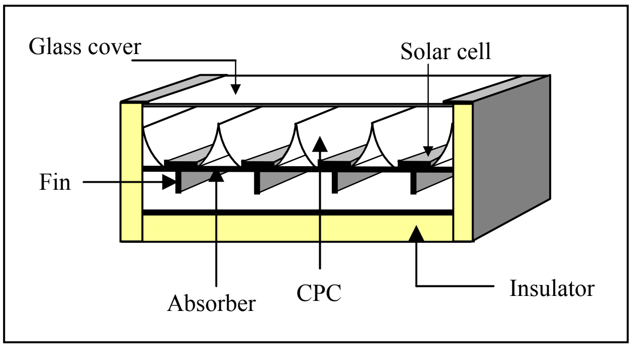


**Figure-1.** Design of both sides single pass PVT.

The subsequent mathematical formulation has been written under the following assumptions:

- Air behaves as an incompressible fluid;
- The thermal contact between solar cells and the absorber where they are mounted on, is good enough for not making the distinction between their respective temperature;
- Heat losses are neglected since we assume that both channels are correctly sealed preventing any leakage of air from the collector; and

- The temperatures of the glass cover, solar cells and back plates are vary only in the direction of working fluid flow.

The thermal schematic model of the collector is shown in Figure-2.

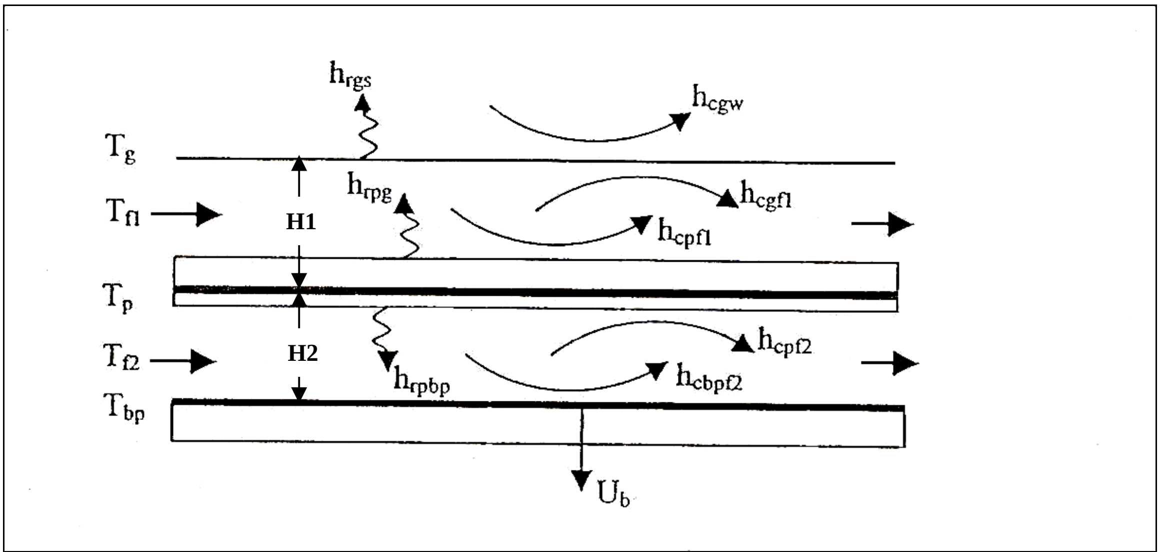


**Figure-2.** Thermal schematic model of the collector.

The thermal energy balance equations for the different nodes of the double pass system are as follows:

### For glass cover

The glass cover gains thermal energy directly by absorbing solar radiation and thermal losses are due to both irradiative and convective heat transfers which expressed in equation as:

$$S_g\rho_g C_g\frac{\partial T_g}{\partial t}=\alpha_g I_{tot}(CR)(1+\tau_g P_p P_R^{2n'})+h_{rg,s}(T_s-T_g) \qquad (1)$$

$$+h_{cg,w}(T_w-T_g)+h_{cg,f1}(T_{f1}-T_g)+h_{rp,g}\frac{A_{ab(T)}}{A_c}(T_p-T_g)-S_g k_g\frac{\partial^2 T_g}{\partial x}$$

where

$$I_{tot} = I_B + I_D$$

$$CR = \frac{1}{\sin\theta}$$

$$T_s = 0.0552\,(T_a)^{1.5}$$

### For air stream in upper channel

The air entering the upper channel carries heat while moving forward because it removes heat by convection from both glazing and photovoltaic plate. The unsteady state governing equation giving us the upper air stream temperature read therefore as:

$$H_1\rho_{air}C_{air}\frac{\partial T_{f1}}{\partial t}=-\frac{m_{air}C_{air}}{H_1}\frac{\partial T_{f1}}{\partial x}+h_{cg,f1}(T_g-T_{f1})+h_{cp,f1}\frac{A_{ab(T)}}{A_c}(T_p-T_{f1}) \qquad (2)$$

### For absorber plate

The third governing equation comes from the energy balance at the plate separating the two air flows. That plate which includes the photovoltaic cells and the absorber plate are submitted to solar radiation through the glazing on the top and the equation will be as:

$$S_p\rho_p C_p\frac{\partial T_p}{\partial t}=\tau_g\alpha_p I_u(CR)P_R^{n'}d[1+(\frac{P_p P_g P_R^{2n'}}{CR})](1-P)+\tau_g\alpha_{pv}I_u P(CR)P_R^{n'}d[1+(\frac{P_{pv}P_g P_R^{2n'}}{CR})](1-\eta_{pv})+$$

$$h_{cp,f1}\frac{A_{ab(T)}}{A_c}(T_{f1}-T_p)+h_{rp,g}\frac{A_{ab(T)}}{A_c}(T_g-T_p)+h_{cp,f2}\frac{A_{ab(B)}}{A_c}\eta_p(T_{f2}-T_p)+h_{rp,bp}\frac{A_{ab(B)}}{A_c}(T_{bp}-T_p) \qquad (3)$$

where

$$I_u = \frac{(I_B + I_D)}{CR}$$

$$\eta_p = 1 - \frac{A_{fin}}{A_{ab(B)}}(1-\eta_f)$$

$$\eta_f = \frac{\tanh\ mh_f}{mh_f}$$

$$\eta_{pv} = \eta_{ref} * (1 - 0.0054\,(T_{pav} - T_{ref}))$$

$$m = \frac{2h_c}{k_f w_f}$$

where $\eta_{ref}$ is the reference efficiency of solar cell at $T_{ref}$ =25 $C^0$

### For air stream in lower channel

Applying the same analysis as we did for the upper air stream the equation is:

$$H_2\rho_{air}C_{air}\frac{\partial T_{f2}}{\partial t}=\frac{m_{air}C_{air}}{H_2}\frac{\partial T_{f2}}{\partial x}+h_{cbpf2}(T_{bp}-T_{f2})+h_{cpf2}\frac{A_{ab(B)}}{A_c}\eta_p(T_p-T_{f2}) \qquad (4)$$

### For back plate

The back plate gets heated by being radiated from the above photovoltaic plate; the air flowing along the lower channel removes a part of its thermal energy owing to its vigorous mixing. Moreover, in spite of insulated sheet covering the bottom duct, heat losses to the ambient have to be taken into consideration. And equation can be stated as:

$$S_{bp}\rho_{bp}C_{bp}\frac{\partial T_{bp}}{\partial t}=U_b(T_a-T_{bp})+h_{cbp,f2}(T_{f2}-T_{bp})+h_{rp,bp}\frac{A_{ab(B)}}{A_c}(T_p-T_{bp}) \qquad (5)$$

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The convection heat transfer coefficients were obtained from Ong [8] obtained the ratio of the Nusselt number for rough and smooth duct for the flat sheet type as follows:

$$\frac{Nu_{rough}}{Nu_{smooth}} = 1.101 + 8*10^{-6} R_e - 5*10^{-11} R_e^2$$

$$Nu_{smooth} = 0.0158 * R_e^{0.8}$$

$$R_e = \frac{\rho * V * Dh}{\mu}$$

$$h_c = \frac{N_u * k}{Dh} \quad \& \quad Dh = \frac{4A_c}{P}$$

This correlation can be utilized to calculate the convective heat transfer between the absorber plate and the back plate as well as the glass cover.

The radiative heat transfer coefficients between two parallel plates are given as:

$$h_{rg,s} = \sigma\, \varepsilon_g (T_g^2 + T_s^2)(T_g + T_s)$$

$$h_{rp,g} = \frac{\sigma (T_p^2 + T_g^2)(T_p + T_g)}{\frac{1}{\varepsilon_p} + \frac{1}{\varepsilon_g} - 1}$$

We will utilize the same second expression to find the radiative heat transfer coefficient between absorber plate and back plate.

## NUMERICAL PROCEDURE

The space interval $[0,L]$ is decretized by the $Nx + 1$ following points $xj = jdx$ where $j = 0,1,\ldots,Nx$ and $dx=L/Nx$.Similarly, time interval $[0,T]$ is divided by the instants $t_n = ndt$ where $n = 0,1,\ldots, Nt$ and $dt = T/Nt$.

The implicit Gear numerical scheme which is:

$$\left(\frac{\partial T}{\partial t}\right)^{n+1} = \left(\frac{3T^{j,n+1} - 4T^{j,n} + T^{j,n-1}}{2\partial t}\right)$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T^{j+1,n+1} - 2T^{j,n+1} + T^{j-1,n+1}}{\partial x^2}$$

Applying this formula for example for the air stream in upper channel we will get:

$$T_{f1}^{j,n+1}\left[\frac{3H_1\rho_{air}C_{air}}{2\partial t} + h_{cg,f1}^{j,n+1} + h_{cp,f1}^{j,n+1}\frac{A_{ab(T)}}{A_c}\right] + T_g^{j,n+1}(-h_{cg,f1}^{j,n+1}) +$$

$$T_p^{j,n+1}\left(-h_{cp,f1}^{j,n+1} * \frac{A_{ab(T)}}{A_c}\right) = \frac{-m_{air}C_{air}}{2H_1}\left(\frac{T_{f1}^{j+1,n} - T_{f1}^{j-1,n}}{\partial x}\right) + \frac{H_1\rho_{air}C_{air}}{\partial t}\left[2T_{f1}^{j,n} - \frac{1}{2}T_{f1}^{j,n-1}\right]$$

With $j = 1,2,\ldots,N_x$-1 and $n\ \varepsilon\ \{1,2,\ldots,N_t\text{-}1\}$. In addition to these $N_x$-1 equations, we write below two equations which are approximated expressions of the boundary conditions that $T$ must satisfy at $x = 0$ and $x = L$.

$$-T^{0,n+1} + T^{1,n+1} = 0$$

$$-T^{J\max -1,n+1} + T^{J\max,\ n+1} = 0$$

$$T_{f1}(0,t) = T_{f2}(0,t) = T_{meassured}\quad (t)$$

Proceeding in that way for Eqs. (1), (3), (4) and (5), one obtains a non-linear system the form of which can be written as:

$$A^{n+1} I^{n+1} = B^{n+1} \qquad (6)$$
$$I^0 = I_0$$

where

$A^{n+1}$ is a square band matrix of size 5 ($N_x$+1) to be inverted. That matrix contains elements which require the knowledge of $T_1$, $T3$ and $T_5$ taken at time $t_{n+1}$ and at different locations $x_j$; Hence the superscript notation used.

$I$ is the vector (size 5($N_x$+1)x1) containing the unknown temperatures

$$T_1^{0,x+1},\ldots,T_1^{N_x,n+1},T_2^{0,n+1},\ldots,T_2^{N_x,n+1},\ldots,T_5^{0,n+1},\ldots,T_5^{N_x,n+1}$$

$I_0$ Contains the known initial conditions of our problem namely

$$T_1^{0,0},\ldots,T_1^{N_x,0},\ldots,T_2^{0,0},\ldots,T_2^{N_x,0},\ldots,T_5^{0,0},\ldots,T_5^{N_x,0}$$

$B$ is the right hand side vector of the system (6) to solve. It has the same size as the unknown vector and contains some terms which require the knowledge of $T_1$, $T3$ and $T_5$ taken at time $t_{n+1}$ and at different locations $x_j$ ;

## SOLUTION PROCEDURE

Because of its non-linearity, system (6) is solved by using an iterative numerical procedure below described.

- The temperatures ( $T_g$, $T_{f1}$, $T_p$, $T_{f2}$, $T_{bp}$ ) at any point during time n = 0 and n = 1 are initially guessed
- Calculate the radiative and convection heat transfer coefficients according to the initially guessed temperature values.
- Calculate ( $T_g$, $T_{f1}$, $T_p$, $T_{f2}$, $T_{bp}$ ) at any point at time n = 0 and iteration k = 1.
- Compare all calculation temperatures with guess temperatures. If the different between them less than 0.001 end the iteration and calculate for next time. If not use this result as the initially guess and repeat the iteration at the same time n = 0.

The computer programme based on FORTRAN is outlined in the flowchart (Figure-3).

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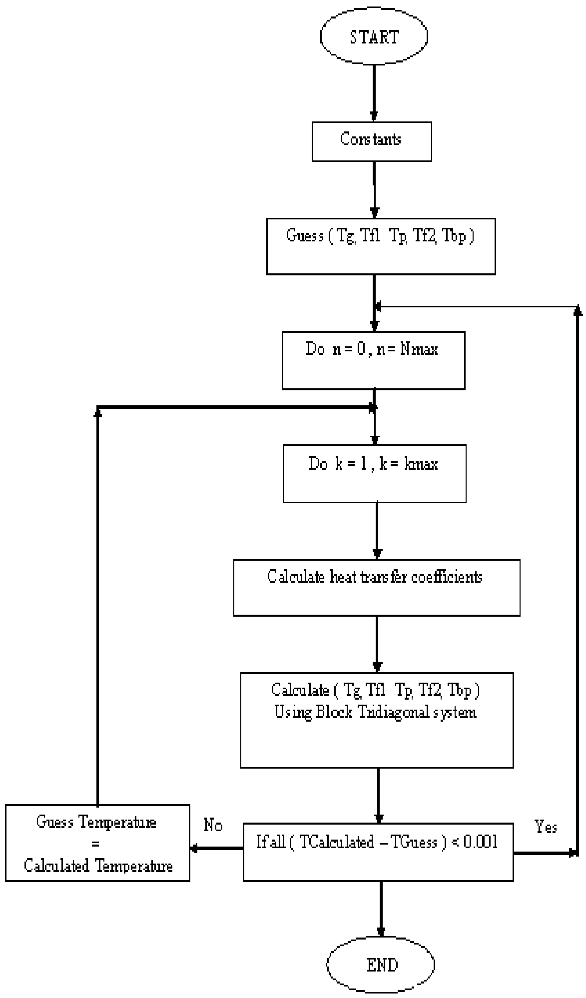


**Figure-3.** Flowchart for computer programme.

The algorithm for this programme is as follows:

1. n = 0
2. n → n+1
3. k = 0
4. k → k+1
5. Calculation of $I^{n,k} = \left[A^{n,k-1}\right]^{-1} B^{n,k-1}$
6. If $k = 1$ then go back to step 4, otherwise proceed.
7. If $\left|I^{n,k} - I^{n,k-1}\right| \leq \in$ is true then:
 (i) $I^{n,0} \rightarrow I^{n,k}$;
 (ii) go back to step 2 if and only if $t_n \neq T$ otherwise go forward to step 8. If the previous inequality is wrong, then go back to step 4.
8. Numerical procedure completed.

$\left[A^{n,k-1}\right]^{-1}$ is computed by using Gauss-Seidel algorithm which is solving using Block-Tridiagonal system.

**PERFORMANCE PARAMETERS**

The efficiency of the combined photovoltaic thermal collector is defined as the sum of thermal efficiency and electrical efficiency as:

$$\eta_{pv/t} = \eta_{\text{thermal}} + \eta_{\text{electrical}}$$

The thermal efficiency of the collector with CPC and fins is as follows:

$$\eta_{th} = \frac{m^{\cdot}\ c_p \int (T_0 - T_i)\, dt}{A_c\ CR \int I_{tot}\, dt}$$

The electrical efficiency is as follows:

$$\eta_{elect} = \frac{\int \tau_g\, I_u\, \eta_{pv}\, \alpha_{pv}\, P\, CR\, d\, \rho_R^{n'} \left[1 + \left(\frac{\rho_{pv}\, \rho_g\, \rho_R^{2n'}}{CR}\right)\right] dt}{CR \int I_{tot}\, dt}$$

The solar cell efficiency is as follows:

$$\eta_{pv} = \eta_{ref} \left[1 - 0.0045 \left(T_{pav} - T_{ref}\right)\right]$$

Where the $\eta_{ref}$ is the reference efficiency of the solar cell at $T_{ref}$ = 25 °C which is in our study 15 %.

**RESULTS AND DISCUSSIONS**

The effect of the mass flow on the efficiencies (photovoltaic, thermal, and combined pv/t) of the collector was shown in Figure-4. The results show that, when the collector is operating at high mass flow rate; the efficiencies (photovoltaic, thermal, and combined pv/t) will increase. This is expected when the photovoltaic panel is cooled by the incoming air. As seen in Figure-4. The photovoltaic efficiency is 11.4% to 12.7% at solar radiation of 400W/m$^2$ and inlet temperature of 30°C. The thermal efficiency is 15.2% to 26.4%. The combined pv/t efficiency is 26.6% to 39.13%. We can see that the combined pv/t efficiency is decrease at low flow rate because the mean photovoltaic temperature is high. Therefore, cooling of the photovoltaic cells by increasing the mass flow rate will increase the combined photovoltaic thermal efficiency.

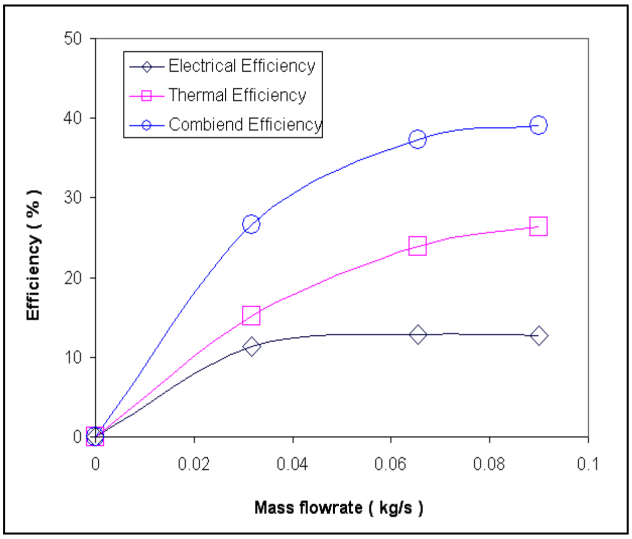


**Figure-4.** The effect of mass flow rate on efficiencies at 400W/m$^2$ and Ti = 30°C.

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**CONCLUSIONS**

The simulation model can predict the performance of the system such as air out let temperature, electrical, thermal and combined efficiencies for different mass flow rates, different solar radiation and different inlet temperatures. It is expected that the prediction results will agreed with the experimental results which will taken in our lab. We conclude that once the developed simulation model is successfully verified by experimental results, this model will useful at different operating conditions such as flow rates, inlet temperatures, sizing and so on.

**NOMENCLATURE**

| | |
|---|---|
| A | Surface area (m2) |
| C | Specific heat (J kg $^{-1}$ K$^{-1}$) |
| CR | concentration ratio of CPC = 1.86 |
| d | Gap loss correction |
| h | Heat transfer coefficient (Wm$^{-2}$ K$^{-1}$) |
| H | Height between glazing / PV-plate or between PV-plate / bottom plate (m) |
| I | Solar irradiance (Wm$^{-2}$) |
| $\dot{m}$ | Mass flow rate (kg s$^{-1}$) |
| n | average number of reflection for radiation passing through CPC inside the acceptance half angle |
| P | Solar cell packing factor |
| Re | Reynolds number |
| S | Thickness (m) |
| T | Temperature (K) |
| $T_s$ | Equivalent black body sky temperature (K) |
| *t* | Time variable (s) |
| $U_b$ | Heat loss coefficient (Wm$^{-2}$ K$^{-1}$) |
| V | Velocity of the air moving through the channels (m/s) |
| W | Collector width (m) |

**Subscripts**

| | |
|---|---|
| a | Ambient |
| ab(T) | Top absorber surface |
| ab(B) | Bottom absorber surface |
| c | Convective |
| P | Reflector |
| r | Radiative |
| tot | Total |
| g | Glass cover |
| $f_1$ | Working fluid (air) at first channel |
| p | Absorber plate |
| $f_2$ | Working fluid (air) at second channel |
| bp | Back plate |
| j | refers to position $x_j$ |
| n | refers to time $t_n$ |
| k | refers to iteration No. k |

**Greek letters**

| | |
|---|---|
| $\rho$ | density (kg m$^{-3}$) |
| $\lambda$ | Thermal conductivity (Wm$^{-1}$ K$^{-1}$) |
| $\alpha$ | Absorptivity |
| $\eta$ | Efficiency |
| $\tau$ | Transmitivity |
| $\varepsilon$ | Emissivity |
| $\sigma$ | Stefan-Boltzmann constant |

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