VOL. 3, NO. 1, FEBRUARY 2008 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

BUFFER STOCKS IN KANBAN CONTROLLED (TRADITIONAL) UNSATURATED MULTI-STAGE PRODUCTION SYSTEM

M. Sreenivasa Rao[1], K. V. Sharma[1], A. Seshu Kumar[2] and G. Krishna Mohana Rao[1]
[1]Department of Mechanical Engineering, JNTU College of Engineering, Kukatpally, Hyderabad, India
[2]Indian Institute of Chemical Technology, Hyderabad, India
E-mail: kmrgurram@rediffmail.com

ABSTRACT

In recent years, most of the manufacturers are resorting to Kanban mechanism to control Work in Process inventories (WIP) in their production activity. Typical production system faces various sources of uncertainties, viz, external demand uncertainty, processing time variations and yield uncertainty etc and Kanban systems are not an exception to this. Often safety stocks are deployed to reduce the ill effects of uncertainties. In literature, there exist insights on the issues relevant to deployment of safety stocks in a Kanban controlled production system however under the assumption that external demand for the finished product is very large also known as saturated condition. In the present global competition, most products have a finite demand rate and stochastic in nature known as unsaturated condition. The present paper deals with Kanban controlled multi-stage unsaturated production (traditional) system with multiple sources of uncertainties in external demand and processing times. For the considered system, Continuous Time Markov Chain (CTMC) model is developed and expressions for steady state performance measures, viz, Probability that a customer demand is satisfied instantaneously on his arrival (known as Customer Service Level), average inventory and average throughput are derived. The expressions are subjected to numerical experimentation and to gain insights for evaluating the performance of the system. The analytical results are validated with the results obtained from stochastic discrete event simulation model at 95% Confidence Level (C.F). Further certain guidelines on the deployment of safety stocks in the considered system are presented.

Keywords: production, system, buffer stocks, kanban, confidence level.

1.0. INTRODUCTION

In recent years**,** most of the manufacturers are in the process of implementing Lean manufacturing control in their production process to reduce inventories. It is commonly implemented through the use of Kanbans and therefore it is generally called as Kanban control mechanism. Many models have been developed for gaining insights for the design of such control mechanism (Monden [1981 a, b, & c, 83, and 84], Altiok and Ranjan [1995], Altiok [1996] and Tayur [1993]) however they are empirical in nature. They are developed with the assumption that external demand for the finished product is very large and referred to as saturated condition through simulation methods. In the present global market scenario, external demand for most of the products is finite or under unsaturated condition and stochastic in nature.

Various sources of uncertainties, viz, external demand uncertainty, processing time variations, yield uncertainty are common in any typical production system. Kanban controlled production (KCP) systems are not an exception to this. Often, safety stocks are deployed to reduce the ill effects of uncertainties. If safety stocks are used in a multi-stage production system, determination of their location and sizing are the issues. The present work is directed towards determining the number of Kanbans to be allocated at each stage of the system such that a specified performance is achieved with minimum average inventory. Under the assumption of saturated condition, Tayur [1993] developed a heuristic based on sample path analysis for the determining the number of Kanbans to be specified at each stage of KCP system such that specified throughput is achieved with minimum average inventory.

In the present paper, unsaturated KCP (traditional) system with sources of uncertainties in external demand and processing times is considered. For the considered system, Continuous Time Markov Chain (CTMC) model is developed and expressions derived for various performance measures namely, probability that external demand is satisfied instantaneously (here after called as Customer Service Level (CSL), Average Inventory (AI) and average number of finished products produced per unit time period (called Average throughput-THT). With the aid of numerical results, insights on the issues relevant to the deployment of safety stocks in the considered system such that specified CSL is achieved with minimum AI is presented.

2.0. MODEL

Figure-1 shows the schematic representation of Kanban controlled mechanism (traditional) for unsaturated two-stage serial production system. In each stage k (k = 1 and 2) consists of queues of output buffer (OB_k) and Kanban Post (KP_k). The entities of queues OB_k and KP_k are finished products attached with Kanbans and Kanbans without attached to parts (hereafter, these are called as free Kanbans) respectively. The M_k and QD represent the machine in stage k (k = 1 and 2) and the queue with entities as backorders respectively. Let c_k be the number of Kanbans associated with the stage k. The sum of entities in the queues in OB_k, KP_k and on M_k at any time is equal to c_k. The total number of Kanbans deployed in the system

VOL. 3, NO. 1, FEBRUARY 2008 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

(C) is equal to $\sum_{k=1}^{2} c_k$. The following are the assumptions made in the analysis.

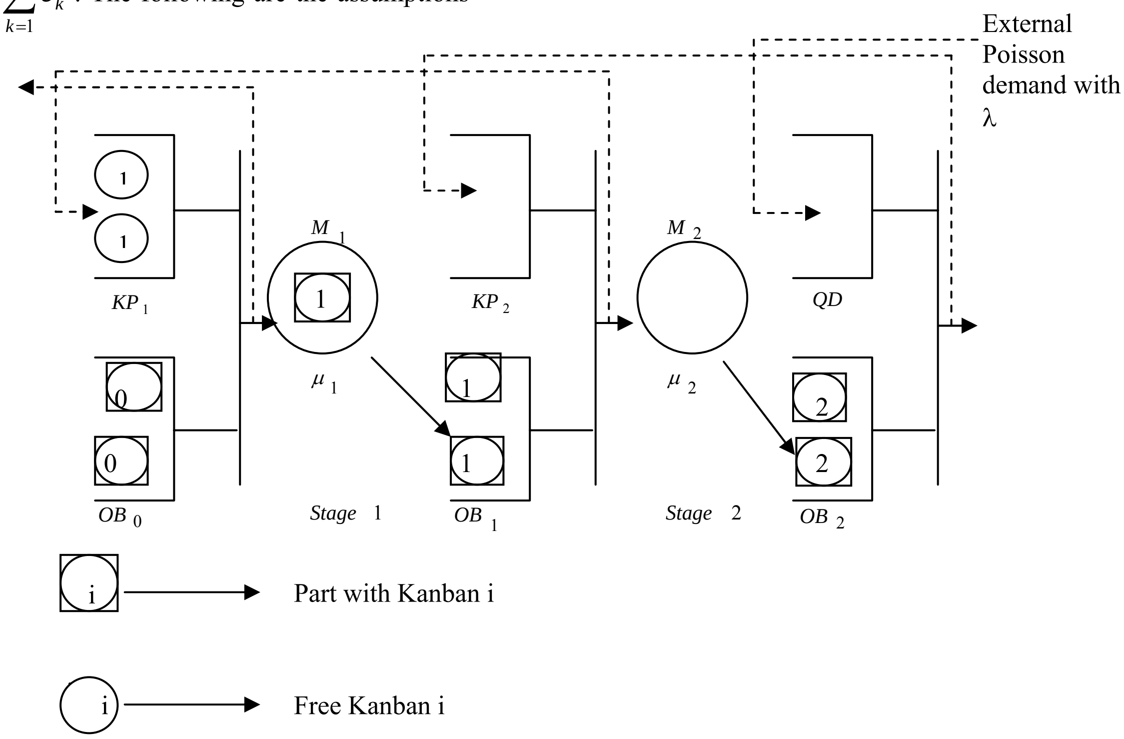

Figure-1. Schematic representation of two-stage unsaturated KCP (traditional) system.

2.1. Assumptions

- M_k processes one part at a time and its processing time is random and modeled as exponentially distributed with a mean processing rate μ_k .
- External demand exists only for the finished product of stage 2 and finite, essentially known as unsaturated condition. The external demand arrival process is assumed to be stochastic.
- The random process involved in external demand arrival rate is assumed to follow Poisson distribution with a mean arrival rate, λ .
- When external demand arrives, it is satisfied immediately if the finished product is available. Otherwise, the demand is backordered / backlogged till the product is ready.
- Let B be the maximum number of backorders allowed in the system and it is assumed that a demand is lost if it arrives when specified number of backorders are already accumulated in the system.
- Let QD be the queue where backlogged demand waits for the finished product of stage 2.
- It is assumed that stage 1 of multi-stage production system has infinite raw material supply.

The dynamics on time scale, called the 'Law of Motion' associated with the system is explained below.

2.2. Law of motion

In the considered unsaturated Kanban system, the possible events that can change the state of the system are the arrival of an external demand for the finished product of stage 2, completion of service by the machine M_k (k = 1 and 2), part departure on completion of operation from the machine M_k and arrival of free Kanban to KP_k (k = 1 and 2). The changes in the state of the system on occurrence of these events can be described as follows:

- When an external demand arrives it is satisfied immediately if at least one finished product is available in the output buffer OB_2, resulting in the assembly of product with demand, termed as unsaturated condition. The finished product leaves the system along with external demand after its associated Kanban is detached without any time delay. The detached Kanban joins KP_k instantaneously. On the other hand, if there are no finished products in OB_2 and the number of backorders accumulated in the queue QD is less than a specified number of backorders B, the external demand joins the queue QD and remains there until a finished product joins the output buffer OB_2. On the arrival of an external demand, if the backorder level in the queue QD is B, then the demand is assumed to be lost, since finite numbers of backorders are allowed. Hence, the queues OB_2 and QD can never be non-empty simultaneously although they can be empty simultaneously.

VOL. 3, NO. 1, FEBRUARY 2008 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

- When the machine M_k completes an operation on a part, the part along with Kanban is added to the output buffer OB_k without any time delay. The machine M_k stops processing and the stage *k* is said to transit to blocking state if KP_k is empty. The stage k continues in blocking, *b* state, till a free Kanban joins KP_k. On the other hand if KP_k is non-empty and if there are no parts in OB_{k-1}, the stage k is said to transit to starvation state. The stage k (k = 1 and 2) remains in that state until a part is transferred to OB_{k-1}. It is to be noted that stage 1 never transits to starvation state as it is assumed to have infinite supply of raw parts.
- When a part along with its tagged Kanban departs from M_k to output buffer OB_k, the part is detached from its associated Kanban and attached with a free Kanban from KP_{k+1} without any time delay if the stage k+1 is in starvation state. The machine M_{k+1} start processing the part. The Kanban which is removed from the part is transferred to the KP_k instantaneously. Otherwise the part remains in OB_k.
- On the arrival of free Kanban to KP_k if stage k is in blocking state and at least one part is available in OB_{k-1}, it is tagged with a part after removing its already associated Kanban of stage $k-1$ without any time delay. The removed Kanban joins KP_{k-1} instantaneously. The Kanban along with attached part moves to stage k and is loaded on to the machine M_k. Otherwise the free Kanban remains in KP_k. Hence queues KP_k and OBk_{-1} can be non-empty simultaneously in the considered system, typical characteristic of traditional system.

As the external demand arrival process follows Poisson distribution, machine service times undergo exponential variation and system assumes the discrete state at any instant, the underlying stochastic process of the system is continuous time Markov chain (CTMC) process. Hence, the system can be appropriately analyzed using CTMC analysis procedure.

3.0. CTMC modeling and analysis

As each system consists of only two stages, exact analysis is carried out by dealing with the underlying Markov-chain. Let A (t), B (t) and J (t) be respectively the number of entities in the queue OB_1, the status of machine M_2 and the difference in the number of entities in the queues OB_2 and QD at time instant t. Using A (t), B (t) and J (t), the state of the system at any time instant t (S (t) can be defined as:

$$S(t) = \{A(t),\ B(t),\ J(t);\ t \geq 0\} \quad (1)$$

The state-space of each of these random variables, viz., A (t), B (t) and J (t) can be represented as:

$$S(A(t)) = \{0,1,2,.....c_1\} \quad (2)$$

$$S(B(t)) = \{\text{idle}(0), \text{busy}(1)\} \quad (3)$$

$$S(J(t)) = \{-B, -B+1,.....0.1.2,....c_2\} \quad (4)$$

Figure-2 represents the state-space diagram of underlying CTMC of the system for given values of c_1, c_2 and B. Let N be the total number of states possible in state-space of the system. By using geometry of the state-space diagram, the expression for N can be obtained as:

$$N = (c_2 + B + 1)(c_1 + 2) - 1 \quad (5)$$

3.1. Proposition

The state-space of the two-stage unsaturated KCP (traditional) system is irreducible and aperiodic CTMC process.

3.1.1. Proof

It can be observed from Figure-2 that no system state is of absorbing type. As λ and μ_k (k = 1 and 2) assumes finite values for the considered system, any state can be reached from any other state in a finite amount of time. Therefore the state-space of the two-stage unsaturated KCP system is irreducible and aperiodic CTMC process. Based on the proposition, each state has independent steady-state probability exist. Using steady-state CTMC analysis, independent steady-state probabilities of all states are determined and by proper summing up of steady-state probabilities, expressions are derived for steady-state performance measures viz., CSL, AI, and THT.

ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

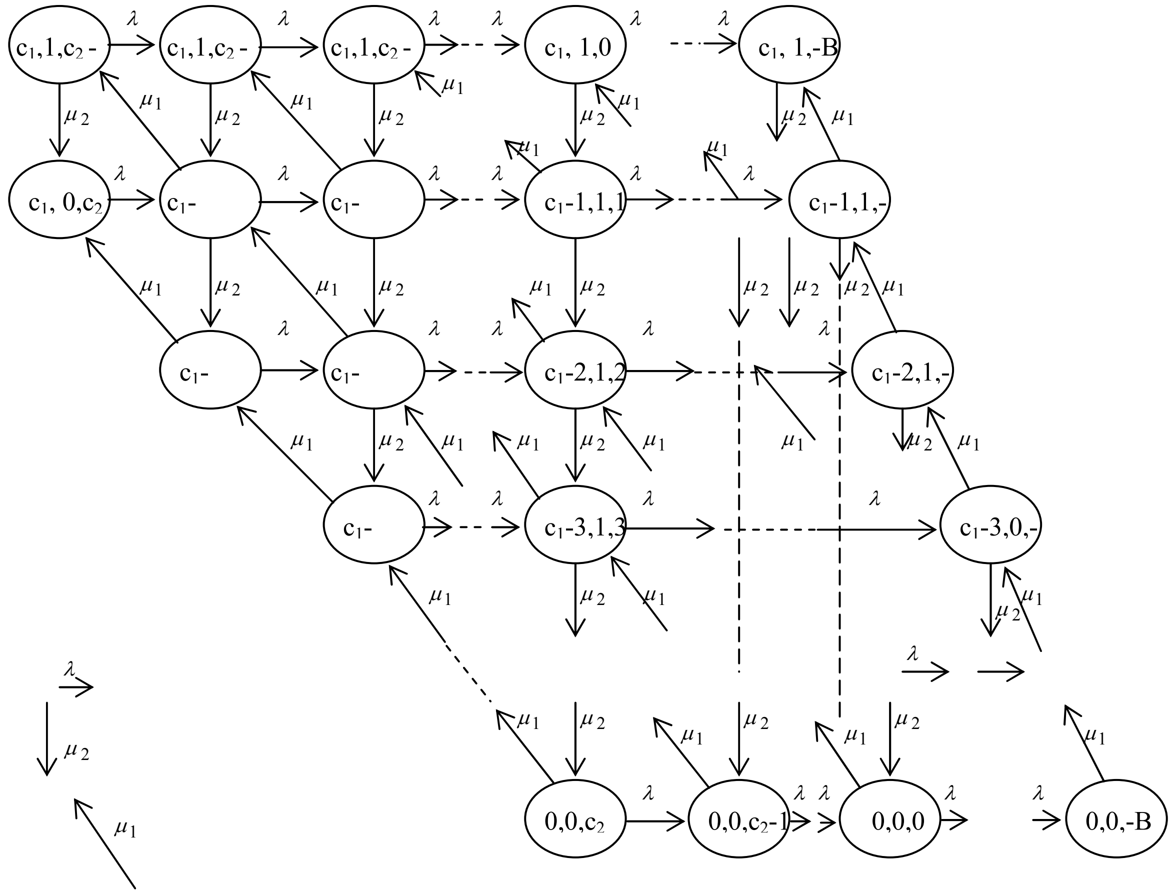

Figure-2. State-space diagram for CTMC of two-stage unsaturated KCP system.

3.2. Steady-state CTMC analysis

For given values of λ and μ_k (k=1 and 2), the transition rate (*Q)* matrix of size $N \times N$ of the system is presented in Rao [2006]. The steady-state probability vector $\Pi = \{\pi(c_1, 0, c_2), \pi(c_1-1, 0, c_2) \ldots \pi(0, 0, -B)\}$ can be obtained by solving the following linear equations (Viswanatham & Narahari [1997])

$$\Pi Q = \underline{0} \qquad (6)$$

$$\Pi\ \underline{e}\ = 1 \qquad (7)$$

where $\underline{e}$ is a column vector of ones with $N \times 1$ size.

3.2.1. Performance measures

Let (l, m, n) be a typical state in the state-space diagram and π (l, m, n) be the steady-state probability of the state (l, m, n). The expressions for the performance measures viz., customer service level (CSL), average inventory (AI) and Throughput (THT) are derived by proper summing of steady-state probabilities. They are given by:

CSL = {Probability that OB_2 is non-empty} (8)

$$CSL = \sum_{l=0}^{c_1}\left[\sum_{n=1}^{c_2-1}\pi(l,1,n)+\pi(l,0,c_2)\right] \qquad (9)$$

Let AF_k (k = 1, 2) be the average number of free Kanbans at stage *k*. Using definitions for AF_1 and AF_2, their expressions can be written as follows:

$$AF_1 = \sum_{l=1}^{c_1-2}\left[\sum_{n=B}^{c_2-1}\pi(l,1,n)+\pi(l,0,c_2)\right]*\left(c_1-(l+1)\right)+\sum_{n=-B}^{c_2}(c_1-1)*\pi(0,0,n) \qquad (10)$$

$$AF_2 = \sum_{n=2}^{c_2-2}\sum_{l=0}^{c_1}\pi(l,1,n)*(c_2-(n+1))+\sum_{n=1}^{c_2-1}\pi(0,0,n)*(c_2-n)+\sum_{n=-B}^{0}\left[\sum_{l=0}^{c_1}\pi(l,1,n)*(c_2-1)+\pi(0,0,n)*c_2\right] \qquad (11)$$

Using Eqns. (6) and (7), expression for AI can be written as:

VOL. 3, NO. 1, FEBRUARY 2008 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

$$AI = (c_1 + c_2) - \sum_{k=1}^{K} AF_k \quad (12)$$

$$THT = \mu_2 \, [1 - \text{Prob. That machine } M_2 \text{ is idle}] \quad (13)$$

$$THT = \mu_2 \left[1 - \left[\sum_{l=1}^{c_1} \pi(l,0,c_2) + \sum_{n=-B}^{c_2} \pi(0,0,n) \right] \right] \quad (14)$$

4.0. RESULTS AND DISCUSSIONS

The unsaturated KCP system is evaluated for the selected performance measures, viz. CSL, *AI* and THT using Eqns. (9), (12), and (14) considering different machine capacities, external demand rates and allocation patterns. To validate the results, stochastic discrete event simulation model (Law and Kelton [2003]) is developed for the system and run with the same inputs. Table-1 shows the validation of CTMC analysis of KCP system with simulation results at 95% confidence level (C.F) when machine capacities are identical. It can be noted that they are in close agreement.

Numerical experiments are conducted to study the following factors on the performance of the system:

a) Effect of variation of mean external demand rate;

b) Effect of allocation patterns; and

c) Effect of additional Kanban.

4.1. Effect of variation of mean external demand

Figures-3 and 4 show the effect of variation of mean external demand rate on CSL and AI of KCP system respectively, when machine service rates are identical. It shows that as the mean external demand rate increases, both CSL and AI decrease. Similar performance of the system is observed even when machines capacities are non-identical also.

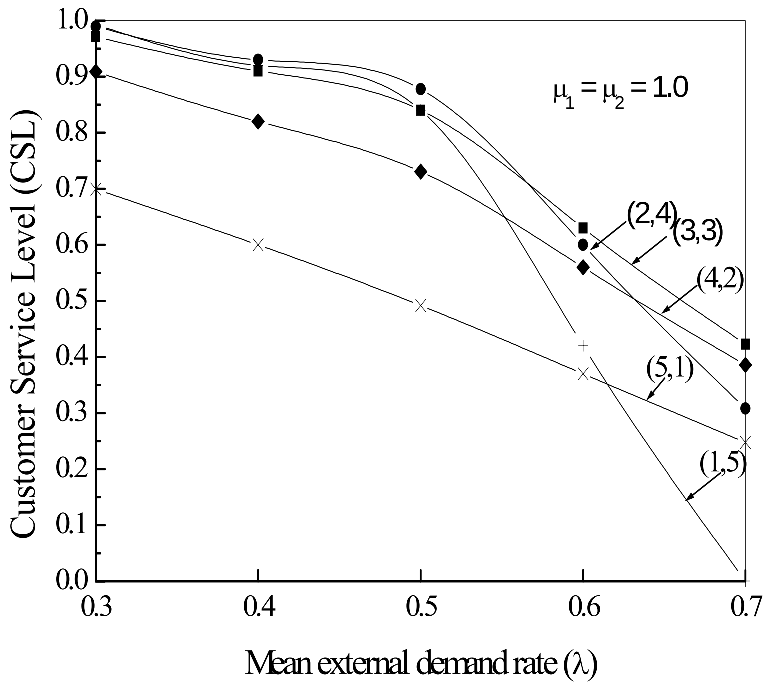

Figure-3. Effect of variation of mean external demand on CSL.

VOL. 3, NO. 1, FEBRUARY 2008 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

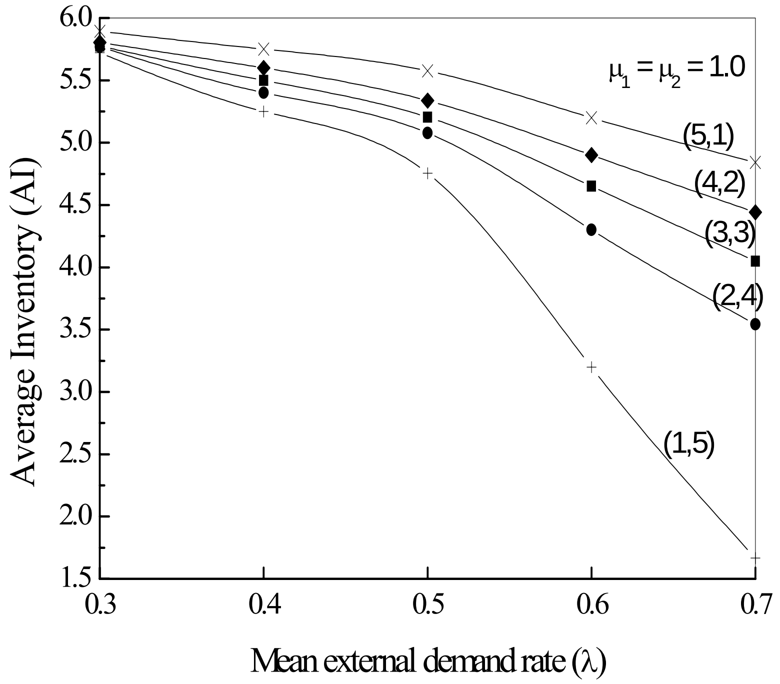

Figure-4. Effect of external demand rate on AI.

4.2. Effect of allocation pattern

Table-1. Validation of CTMC analysis of two-stage unsaturated KCP (traditional) (TKCP) system with simulation results for the conditions of $\mu_1 = \mu_2 = 1.0$, C=6 and B = 100.

S. No.	Mean Ext. Demand Rate	Allocation	TKCP – CTMC analysis		Simulation with 95% C.F	
			AI	CSL	AI	CSL
1	0.3	3, 3	5.7756	0.97126	5.7760± .0017	0.9713± .007
		2, 4	5.7695	0.98915	6.0± 0.0035	1.0± 0.0001
		1, 5	5.7208	0.9930	5.778± 0.0035	0.9930± 0.0001
		5, 1	5.8928	0.69960	5.8930±0.0023	0.6996± .0001
		4, 2	5.8032	0.9090	5.8030± .0018	0.90930±0.001
2	0.5	3, 3	5.205	0.8400	5.1831±0.0018	0.8390± 0.003
		2, 4	5.077	0.8777	5.0316±0.0135	0.87690±0.003
		1, 5	4.755	0.8400	4.610± 0.0021	0.8396±0.0048
		5, 1	5.5768	0.4917	5.5760±0.0065	0.4907±0.00314
		4, 2	5.3379	0.7304	5.3270± 0.008	0.7296± 0.0037
3	0.7	3, 3	4.0507	0.4230	3.86923±0.03	0.4185± .0116
		2, 4	3.5444	0.3084	3.0672± 0.048	0.3010± 0.017
		1, 5	1.66802	0.0005	1.6670± 0.002	0.0002±0.0003
		5, 1	4.843	0.2477	4.8370± .0189	0.2457±0.0044
		4, 2	4.4423	0.3858	4.3533± 0.023	0.3822± 0.008

To analyze the effect of allocation pattern on the performance of the system, possible allocations with given total number of Kanbans as six is considered and for each allocation the performance of the system with respect to CSL and AI is evaluated using Eqns. (9) and (12) at different external demand rates. Table-1 shows the effect of allocation patterns on the performance of the system at different external demand rates when machine capacities

ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

are identical. It can be observed from the Table-1 that when external demand is $\lambda = 0.3$, allocation pattern of (1,5) yields minimum AI of 5.7208 and maximum CSL of 0.9930 as compared to all other allocation patterns possible with same total number of Kanbans. The system is performing similarly for $\lambda = 0.5$ also. For $\lambda \geq 0.7$, no particular allocation pattern is dominant. At lower rates of demand, the rate at which free Kanbans accumulates in the Kanban post of stage 2 is low. This implies that an allocation of one Kanban to stage 1 can make a part available in its buffer at times required by stage 2. Hence, allocation of more than one Kanban at stage 1 will only increase AI without any further increment in CSL. On the other hand, deployment of more number of Kanbans at stage 2 would improve the availability of finished products in OB_2 and there by CSL also improves. On the other hand, as external demand increases, in KCP system, accumulation of more free Kanbans at KP_2 and starvation of stage 2 for a part occurs at high rates of demand with the allocation of one Kanban at stage 1 and remaining Kanbans at stage 2 for a given value of CK. This results in reduction of AI along with CSL.

4.3. Effect of additional Kanban

For analysis of the effect of allocation of an additional Kanban to the already established allocation pattern of one Kanban in stage 1 and remaining Kanbans to stage 2 on the AI and CSL has been carried out. An additional Kanban has been added to stage 1 and stage 2 alternatively and different allocation patterns are generated. Then, for each generated allocation pattern, AI and CSL are determined at different external demand rates and mean machine service rates and presented in Table-2 for two-stage unsaturated KCP system. From this Table-2, it can be observed that addition of a Kanban to the present allocation will increase CSL but AI also increases in both systems. Up to $\lambda \leq 0.5$, allocation of additional Kanban to stage 2 would increase more CSL with minimum increment in AI as compared to allocation of Kanban to stage 1. For higher rates of external demand ($\lambda > 0.5$), it can be observed that allocation of an additional Kanban either to stage 1 or stage 2 for maximum improvement in CSL with minimum increment in AI, no particular allocation is dominant in KCP system.

Table-2. Effect of an additional Kanban on the performance of two-stage unsaturated KCP (traditional) (TKCP) system when C= 6 and B = 100.

S. No.	Machine Service Rates	Ext. Demand rate	Performance of TKCP with present Kanban Allocation			Performance of TKCP with additional Kanban Allocation		
			Allocation	AI	CSL	Possible Allocation	AI	CSL
1	$\mu_1 = 1.0$ $\mu_2 = 1.0$	0.3	3, 3	5.7756	0.971	3, 4	6.7661	0.9913
						4, 3	6.7754	0.973
		0.5	3, 3	5.205	0.840	3, 4	6.0969	0.9115
						4, 3	6.1874	0.8585
		0.7	3, 3	4.0507	0.423	3, 4	4.394	0.5140
						4, 3	4.875	0.512
2	$\mu_1 = 0.8$ $\mu_2 = 1.0$	0.3	3, 3	5.7002	0.945	3, 4	6.6756	0.9893
						4, 3	6.6821	0.9711
		0.5	3, 3	4.7625	0.775	3, 4	5.618	0.85649
						4, 3	5.7438	0.81665
		0.7	3, 3	2.556	0.044	3, 4	2.6118	0.05649
						4, 3	3.3307	0.1784
3	$\mu_1 = 1.0$ $\mu_2 = 0.8$	0.3	3, 3	5.7002	0.945	3, 4	6.679	0.97904
						4, 3	6.7012	0.9470
		0.5	3, 3	4.9104	0.709	3, 4	5.7164	0.80598
						4, 3	5.921	0.7340
		0.7	3, 3	3.223	0.037	3, 4	3.2717	0.4889
						4, 3	4.2214	0.14137

5.0. CONCLUSIONS

For evaluating the performance of two-stage unsaturated KCP(traditional) system when there exist uncertainties in external demand and processing times, CTMC model is developed and derived expressions for CSL, AI and THT. The analysis is validated with stochastic discrete event simulation model at 95% C.F. Based on the study; following are the conclusions about the performance of unsaturated KCP system:

ARPN Journal of Engineering and Applied Sciences

- The analytical results are in close agreement with simulation results.
- As external demand increases, both CSL and AI decrease at all machining conditions.
- At lower rates of external demand, an allocation of one Kanban at stage1 and remaining Kanbans at stage 2 out of given total number of Kanbans is dominant for the criteria of achieving specified CSL with minimum AI.
- At higher rates of external demand, no particular allocation pattern is dominant.
- An addition of Kanban to the existing allocation improves the *CSL* however with increase in *AI* Further, adding of Kanban to stage 2 would improve more *CSL* with minimum increase in *AI* as compared to adding it to stage 1 at lower rates of external demand.

REFERENCES

Altiok T. and Ranjan R. 1995. Multi-stage pull type production/inventory systems. IIE Transactions. 27: 190-200.

Altiok T. 1996. Performance analysis of manufacturing systems. Springer Series in operations research. pp.188-336.

Law A. M. and Kelton W. D. 2003. Simulation Modeling and Analysis. Tata McGraw-Hill Publishing Company Ltd., New Delhi.

Monden Y. 1981a. What makes the Toyota production system really Tick? Ind. Engg. 13(1): 36-46.

Monden Y. 1981b. Adaptable Kanban system helps Toyota maintain Just-In-Time Production. Ind. Engg. 13(5): 28-46.

Monden Y. 1981c. How Toyota Shortened supply lot production time, waiting time and conveyance time. Ind. Engg. pp. 22-30.

Monden Y. 1983. Toyota Production System: Practical Approach in Production Management. Industrial Engineering and Management Press, Atlanta, GA.

Monden Y. 1984. A simulation analysis of Japanese Just-In-Time technique (with Kanbans) for a Multiline, Multistage Production System: a comment. Decision Sciences. 15: 445-447.

Rao M.S. 2006. Analytical Investigations on Kanban Controlled Unsaturated Multi-Stage Production System. Ph.D Thesis. JNT University, Hyderabad, India.

Tayur S. R. 1993. Structural properties and a Heuristic for Kanban controlled Serial lines. Management Science 39(11): 1347-1367.

Viswanadham N. and Narahari Y. 1994. Performance modeling of Automated Manufacturing Systems. Prentice-Hall India Ltd. pp. 63-473.