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EFFECT OF PILE CAP SYSTEM ON THE DISTRIBUTION OF BENDING MOMENT OF CAP

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ABSTRACT

The present paper undertakes the analysis of pile-cap system under the assumption of continuo piles-pile-cap connection. The piles are simulated by means of springs of varying stiffness. Nonlinearity of the piles was treated under the assumption of a hyperbolic stress-strain relationship. The pile cap was assumed as plate elements. The analysis conducted in this study is the moment distribution within the cap. Comparison was made for distribution under a constant stiffness and varying stiffness assumptions. The effect of pile cap stiffness on bending moment was demonstrated for the analysis. The distribution of bending moments along the two considered sections for cap thickness of 3m, the piles are assumed to have linear behavior. The bending moments have the lowest value at the ends of sections. The bending moment increased when moving towards the center of cap until reaching the maximum value. For pile thickness of 1.0m to 4.0m, the maximum bending moment was found to be located between center line of the pile cap and edges. The maximum bending moments increased with increase in the thickness of the pile cap. The effect of pile cap thickness on the distribution of bending moment is also presented in this paper.

Keywords: pile cap, springs, stress, strain, relationship, moment, distribution.

INTRODUCTION

The majority of piled foundation consists not of a single pile, but a group of piles, which act in the dual role of reinforcing the soil, and also of carrying the applied load down to deeper, stronger soil strata. Usually, the pile group consists of a group of piles covered by a cap (Fleming, 1985, and Hossein et al., 2007). For structural design if cap, the bending moment developed in the cap section are important. The pile cap is defined as a structural member used to distribute the load to the piles (Bowles, 1988). The flexibility of the pile cap affects individual pile head forces significantly and affects the bending moments and shear forces in individual piles as well, even though the displacement of the pile cap does not vary much (Won et al, 2006). A pure fixed-head condition is seldom achievable in the field, even when a pile group is constrained by a stiff concrete pile cap, because the cap itself rotates (Mokwa, 2003), by assume a pure fixed-head boundary conditions to simulate the pile-cap connection.

The pile cap has reaction which consisted of a series of concentrated loads (piles). The pile cap may or may not spread the load on each pile. When the pile cap distribute an equal magnitude of load on each pile, the following assumption must be satisfied (Bowles, 1988); (1) the pile cap is in contact with the ground, (2) the piles are all vertical, (3) a load is applied at center of pile group, and (4) the pile group is symmetrical. As reported by Henry (1986), the pile cap should be capable of safely carrying the imposed bending moment and shearing forces.

Usually, the pile cap is modeled as a rigid plate based on rigid plate theory that detailed by (Bowles, 1988). For a concentric axial load on the cap, each pile carries and equal amount of load. Comodromos *et al.* (2005) explained the influence of the interaction between the piles of a group fixed in a rigid pile cap on both the lateral load capacity and the stiffness of the group. Three-dimensional frame analysis was studied by Henry (1986) in which the pile cap assumed to be rigid and the effect of soil was neglected.

Poulos and Davis (1980) give details of load-transfer method for analysis of pile in the group. The soil properties required in this method are the relationship of the load transfer and the soil shear strength with the pile movement. The piles were divided into a number of segments and a small tip movement was assumed. This method was extended by Chow (1986) to non-homogenous soil and non-linear soil behavior. The non-linear response of the group was dominated by the non-linear response of individual isolated pile. O'Neill *et al.* (1982) described patterns of measured load transfer in a full size instrumented pile group. Kitiyodom, *et al.* (2002) was developed a simplified method of numerical analysis to estimate the deformation and load distribution of piled raft foundations subjected to vertical, lateral, and moment loads. They used a hybrid model in which the flexible raft is modeled as thin plates and the piles as elastic beams and the soil is treated as springs.

In this paper, undertakes the analysis of pile-cap system under the assumption of continuo piles-pile-cap connection. The piles are simulated by means of springs of a varying stiffness. The pile cap was assumed as plate elements. Made a comparison for moment distribution under a constant stiffness and varying stiffness assumptions. The effect of pile cap thickness on the distribution of bending moment will also be detailed in this paper.

THEORY AND MODELING

This part covers the geotechnical models used to estimate bending moments within the pile cap. The pile

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cap is modeled by means of an elastic plate. The piles are modeled as springs with variable stiffnesses. The non-linear behavior of pile is simulated by a non-linear behavior (hyperbolic load-settlement model). The parameters of this model are obtained from the load-settlement curve of a compressive pile load test.

THE FINITE ELEMENT METHOD (PLATE BENDING METHOD)

Pile cap was modeled as a thin plate by using a thin plate element Desai (1977). Selvadurai (1979) defined a thin plate as body consist of two parallel surface with very small or zero curvature. The thickness of the thin plate is the vertical distance between these two surface, which usually smaller than the other dimensions. The thin plate has a middle surface which are parallel with the two surfaces and between them. The coordinate system and nodal moment are shown in Figure-1.

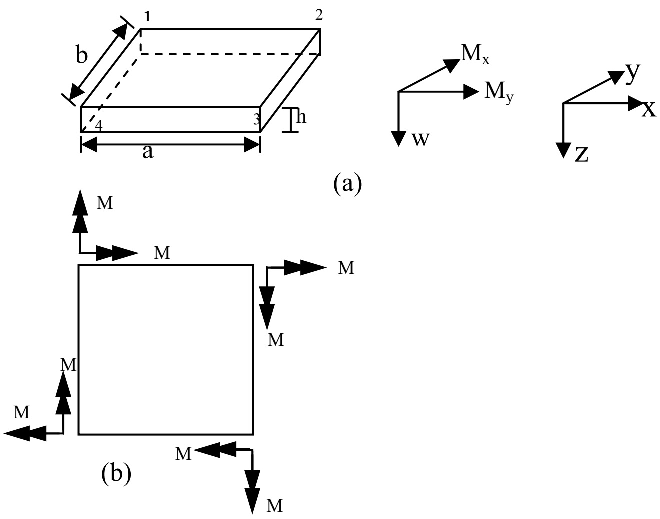

Figure-1. The coordinate system and nodal moment [After Cook, 1974]
a- Coordinate system
b- Nodal moment

This finite element method discretizes the plate into a number of rectangular and/or triangular element. This element is reasonably accurate and it includes the transfer shear deformation effect, and leads directly to a general element. The classical theory of thin plate assumed several factors to satisfy the assumption of thin plates, i.e. the thickness of the plate is small compared to its other dimensions, the deflections are small, the middle plane of the plate does not undergo an plane deformation, and the transverse shear deformation is zero.

In order to study the rectangular plate bending element, the element choses have a 12 degree of freedom (3 d.o.f per node). These degree of freedom represent translation about z-axis, and rotation about two orthogonal axes. Cook (1974) gives the displacement function for plate bending element as follows:

$$\begin{aligned} w = {} & \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 \\ & + \alpha_7 x^3 + \alpha_8 xy^2 + \alpha_9 x^2 y + \alpha_{10} y^3 \\ & + \alpha_{11} x^3 y + \alpha_{12} xy^3 \end{aligned} \tag{1}$$

Zienkiewicz (1977) reported the continuity of lateral displacement and both slopes along an edge of the element. The shape function of a rectangular plate bending element is developed to consist of the parameter which resulted from the derivation of the lateral displacement with respect to both x-axis, and y-axis. The additional term ($\partial^2 w / \partial x \partial y$) represent the bending moment about both two axes. This term increase the degree of freedom per node to (four). Then the total degree of freedom developed in the element is sixteen. The displacement functions of plate bending element.

$$\begin{aligned} w = {} & \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^3 + \alpha_8 xy^2 \\ & + \alpha_9 x^2 y + \alpha_{10} y^3 + \alpha_{11} x^3 y + \alpha_{12} xy^3 + \alpha_{13} xy^2 + \alpha_{14} x^3 y^2 \\ & + \alpha_{15} x^2 y^3 + \alpha_{16} x^3 y^3 \end{aligned} \tag{2}$$

Degrees of freedom at each node are:

$$w, \theta_x = \frac{\partial w}{\partial x}, \theta_y = \frac{\partial w}{\partial y}, \theta_{xy} = \frac{\partial^2 w}{\partial x \partial y} \tag{3}$$

The nodal displacement vector $\{\delta^e\}$ and nodal element forces vector $\{F^e\}$ for the rectangular plate bending element shown in Figure-2 can be written as:

$$\begin{aligned} \{\delta^e\} = \{ & w_1\ \theta_{x1}\ \theta_{y1}\ \theta_{xy1}\ w_2\ \theta_{x2}\ \theta_{y2}\ \theta_{xy2} \\ & w_3\ \theta_{x3}\ \theta_{y3}\ \theta_{xy3}\ w_4\ \theta_{x4}\ \theta_{y4}\ \theta_{xy4}\} \end{aligned} \tag{4}$$

$$\begin{aligned} \{F^e\} = \{ & F_{z1}\ M_{x1}\ M_{y1}\ M_{xy1}\ F_{z2}\ M_{x2}\ M_{y2}\ M_{xy2} \\ & F_{z3}\ M_{x3}\ M_{y3}\ M_{xy3}\ F_{z4}\ M_{x4}\ M_{y4}\ M_{xy4}\} \end{aligned} \tag{5}$$

$$w, \theta_x = \frac{\partial w}{\partial x}, \theta_y = \frac{\partial w}{\partial y} \tag{6}$$

MODELING OF PILES (THE HYPERBOLIC STRESS-STRAIN MODEL)

For analysis of pile group the piles can be modeled as elastic support by replacing the piles by springs. These springs have a constant “k” determined from certain load-settlement curve by using any method which satisfied this model. However, data of this curve are usually determined from pile load test. In general, piles are behaved non-linear and this non-linearity should be satisfied by using reasonable model such as hyperbolic load-settlement model.

The pile reaction, or t-z method referred by Kraft and Kagawa (1981) was obtained by computing the axial movement of a pile under axial load. Kondner (1963) proposed hyperbolic equation to model the stress- strain response of cohesive soil. This equation has two-constants and written as follows:

$$\left(\sigma_1 - \sigma_3\right) = \frac{\varepsilon}{a + b\varepsilon} \quad (7)$$

Where:

$\sigma_1 + \sigma_3 =$ The major and minor principal stresses.

$\varepsilon =$ the axial strains.

$a \,\&\, b =$ constants whose value may be determined experimentally.

ANALYSIS AND RESULTS

The cap considered in this work is a reinforced concrete slab which supports a tower of 208m height. The piles are spaced at 4.0m center to center and embedded to a depth of 37 m. The geometry of the 420-piles cap 83mx79m, and has a thickness of 3m. The pile group is actually a foundation for a "Jama'a Al Kabeer-Big Mosque" in Baghdad, Iraq.

Based on both field and laboratory tests, the soil profile can be described as follows: (1) A fill layer extended to a depth of 2m below the ground level. This layer consists of brown silty clay sometimes with broken bricks. (2) The first main layer extended below the fill layer to a depth of 4.75-7.25m, this layer consists of brown gravel silty clay and clayey silt, sometimes with sand. (3) The second main layer follows the first layer and extended to a depth of 40.5m, this layer consists of grey silty sand, sometimes with clay or fine gravel. (4) The under ground water table is encountered at depth range between 1.5-1.75m below the ground level. Table-1 shows the soil parameter for two layers.

The results of bending moments are calculated at the each node. These nodes represent the location of piles. To make a comparison of results, two section have been considered; one A-A along the x-direction and the other B-B in the y-direction as shown in Figure-2.

Two-dimensional plate bending element method is used to analyze the studied pile group, the pile cap is divided into 380 plate elements. These elements have the same properties and dimensions

The results of bending moments for a cap thickness equal to 3m along two considered sections are shown in Figure-3 and Figure-4. The bending moments start from positive values at nodes located near the edges, and then increased when moving toward the edges until reaching the maximum between the center line of pile cap and the edges.

Positive bending moments at section A-A are equal to the negative bending moments in magnitude and opposite in direction, due to the symmetry of load. Unsymmetrical moment's distribution along section B-B resulted from unsymmetrical load distribution.

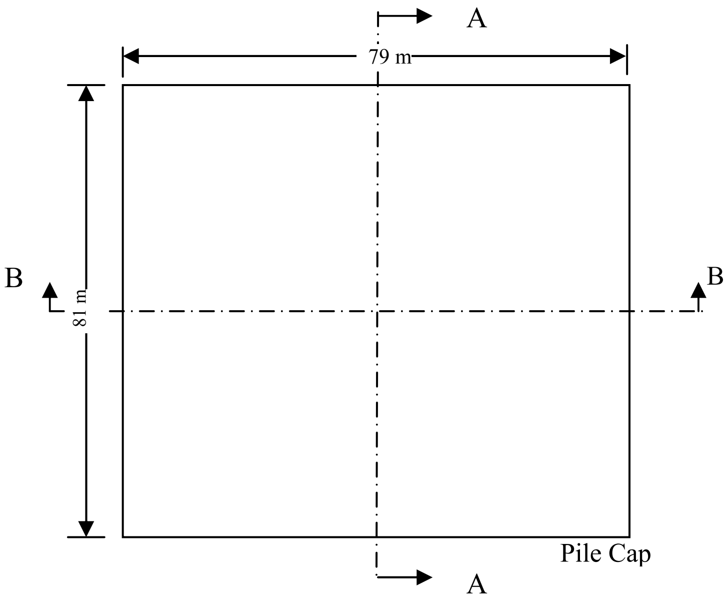

Figure-2. Sections of Pile Cap.

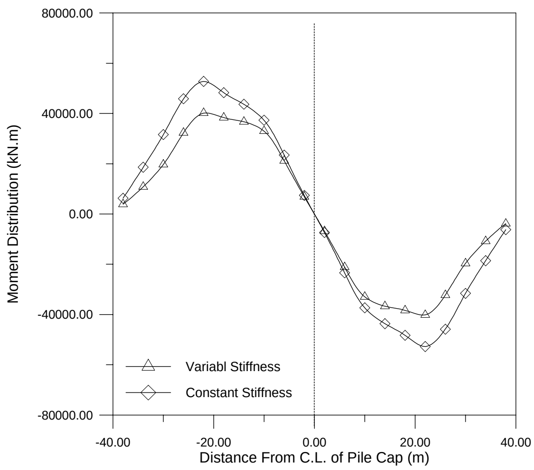

Figure-3. Comparison between moments distribution by plate method along section A-A (Cap Thickness = 3m)

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Table-1. Soil parameters.

Parameter	Unit	First layer	Second layer
Natural water content (Wn)	%	24	23
Natural density (γn)	g/cm^3	1.97	1.82
Dry density (γd)	g/cm^3	1.61	1.58
Spesific gravity (Gs)	_	2.66	2.52
Liquid limit (LL)	%	42	6
Plasticity index (PI)	%	19	2
Cohesion (Cu)	kN/m^3	19.5	20.1
Angle of internal friction (φ)	degree	7.0	32.5

Table-2. Comparison between maximum bending moments.

Bending moment (kN m)	**Linear pile model**		**Non-linear pile model**	
	+ve	-ve	+ve	-ve
Highest bending moment	69357.1	-52770.3	57016.4	-40167.6
Lowest bending moment	6039.4	-5801.8	3555.6	-3145.9

Based on the non-linear behavior of pile, the result of bending moment along the two sections are shown in Figures 3 and 4, the results were calculated for pile cap thickness equal to 3m. The maximum bending moments along the two sections are summarized in Table-2. The linear pile model give a maximum and minimum greater than non-linear pile model (positive and negative bending moment).

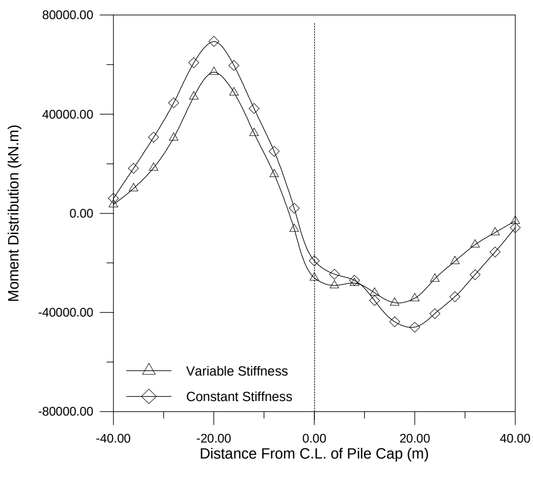

Figure-4. Comparison between moments distribution by plate method along section B-B (Cap Thickness = 3m).

The effect of pile cap thickness on the distribution of bending moment along the two considered section are shown in Figures 5 and 6. These two figures show that bending moment increase with increase the thickness of pile cap. Table-3 consists of maximum bending moment for different pile cap thickness.

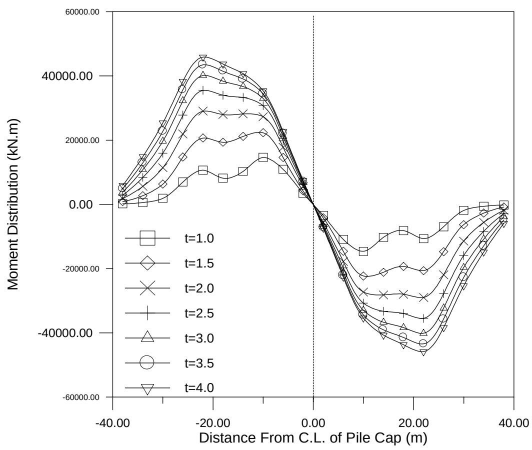

Figure-5. Moments distribution along section A-A.

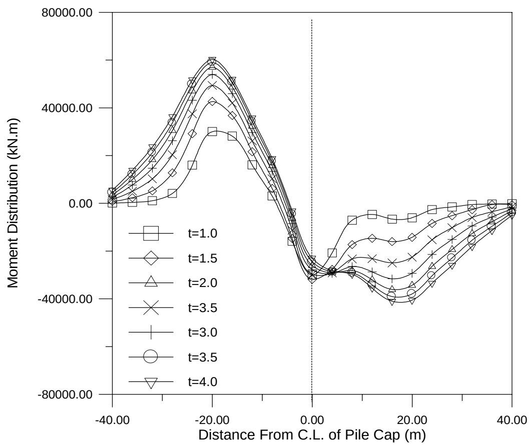

Figure-6. Moments distribution along section B-B.

Table-3. Maximum bending moments.

Cap thickness	Positive bending moments (kN m)		Negative bending moments (kN m)	
	Section A-A	Section B-B	Section A-A	Section B-B
1.0	14627.4	30054.9	-14627.4	-30050.2
1.5	22302.9	42646.3	-22302.9	-31883.2
2.0	28197.6	49388.3	-28197.6	-29773.1
2.5	35518.2	53997.1	-35518.2	-31626.8
3.0	40167.6	57016.4	-40167.6	-36112.8
3.5	43481.6	59034.8	-43481.6	-39085.9
4.0	45865.3	60295.7	-45865.3	-41065.1

CONCLUSIONS

The effect of pile cap system on the behavior of pile group subjected to the axial load has been investigated in the paper through series if 2D finite-element analysis. Base on result theses analysis, the following conclusions can be drawn:

1. The response of pile group in both constant stiffness and variable stiffness pile model is influenced by the thickness of pile cap;
2. The bending moments are effected by the location of piles within the group, the state of loading, and the pile head fixation; and
3. The constant stiffness pile model give bending moments greater than that resulted from the variable stiffness pile model.

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