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DESIGN OF INTELLIGENT HYBRID CONTROLLER FOR SWING-UP AND STABILIZATION OF ROTARY INVERTED PENDULUM

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ABSTRACT

A hybrid controller for swinging up rotary inverted pendulum is proposed in this paper. The controller composes of two parts. The first part is the PD position control to swing up the pendulum from its hanging position by moving the rotary arm clockwise and anticlockwise repeatedly until the pendulum swings up around the upright position. The second part is a Fuzzy Logic Controller which will be switched to balance and stabilize the pendulum at its upright position. State feed back control designed by LQR is also done for stabilization of the pendulum. The proposed intelligent hybrid controller is compared with the conventional controller; the effectiveness and reliability are shown by the simulation results.

Keywords: modeling, fuzzy logic, swing up, PD, switching, control, inverted pendulum, linear quadratic regulator.

1. INTRODUCTION

The Self Erecting Rotary Inverted Pendulum (SERIP) system is a challenging problem in the area of control systems. It is very useful to demonstrate concepts in linear control such as the stabilization of unstable systems. Besides, as a typical nonlinear system, inverted pendulum is often used as a benchmark for verifying the performance and effectiveness of a new control method because of the simplicities of the structure.

Swing up inverted pendulum has many advantages in theoretical study such as simple structure, nonlinear, and uncertain characteristics. Many control methods have been reported in swing up inverted pendulum. For example, swing up using fuzzy control algorithm [1], robust swing up control [2], nonlinear control [3], and energy based methods [4,5]. Astrom and Furuta showed that the global behavior is characterized by the ratio of the maximum acceleration of the pivot to the acceleration of gravity. Though good performance can be obtained by using these algorithms, they are very complicated to implement and their parameters are not easy to design. In [6,7] a fuzzy controller based on single input rule modules (SIRMs) was presented, in which each input term is assigned with a SIRM and a dynamic importance degree. A fuzzy swing up control combined with an LQR stabilizing was proposed, where the influence of disturbance was discussed with an adaptive state controller [8]. Lin and Mon proposed a hierarchical fuzzy sliding controller to achieve decoupling performance [9]. Lam etc. investigated the stability with TSK fuzzy logic associated with feedback gains tuned by genetic algorithm [10]. An observer-based hybrid adaptive fuzzy neural network controller combined with a supervisory controller was presented in [11]. In recent years, hybrid-control, an integration of unique methods, has attracted plenty of attention. In [12], a hybrid PD servo feedback controller designed by pole placement method has been reported. It is known that a control system assigned by pole placement method may not be an optimal one. In another paper [13], a hybrid controller where the part of servo state feedback controller designed by linear quadratic regulator(LQR) problem is proposed for swing up and stabilizing the inverted pendulum on cart. Though it is considered good and robust, being considered a conventional one, the inverted pendulum poses serious problems for qualitative modeling methods, so it is a good benchmark to lest their performance. Our approach to this task consists of deriving control rules from the actions of a human operator stabilizing the pendulum and subsequently using them for automatic control, using fuzzy, an intelligent technique. Rule derivation is based on the "learning from examples" principle and does not require knowledge of a quantitative model of the system.

The control strategy of SERIP system is composed of the swing up control of the pendulum and stabilizing control of the whole system that consists of angular control of the pendulum at upright position and position control of the rotating arm to its initial position while balancing. First of all, swing up control is to bring the pendulum from the downward position to the upright position. This is achieved when the motor is given voltage in the appropriate direction and magnitude to drive the arm to move clockwise and anticlockwise repeatedly until the pendulum is close to the upright position. Thereafter stabilizing control is to balance the pendulum in the upright position.

The remaining part of this paper is organized as follows:

In section 2, the mathematical modeling of the Rotary Pendulum is presented. The positive feedback PD algorithm for swing up process is described in section 3. Section 4 is devoted to the design of stabilizing controller using conventional LQR and Fuzzy technique. The switching control is discussed in section 5. Simulation results are presented in section 6. Finally, section 7 concludes this paper.

2. DYNAMIC MODEL OF THE ROTARY PENDULUM

2.1. Description of the system

The Rotary pendulum system consists of a rotary servo motor system which drives an independent output gear. The rotary pendulum arm of radius ‘r’ is mounted to the output gear and the pendulum of length ‘l’ and mass ‘m’ is attached to the hinge. Clearly, this is an under-actuated mechanical system. The pivot arm of radius ‘r’ rotates in a horizontal plane by the servo motor. The arm must be moved in such a way that the pendulum is in the upright position. The arm rotates in the horizontal plane while the pendulum rotates in a plane that is always perpendicular to the rotating arm as in Figure-1. Here α denotes the angle of the pendulum to the up-right position. and θ denotes the angle of the rotor arm. The angle of the pendulum α is defined to be zero when in upright position and positive when the pendulum is moving clockwise. The angle of the arm θ is positive when the arm is moving in the clockwise direction. The control purpose is to design a controller that starts with the pendulum in the “down” hanging position, swings it “up” and maintains it upright.

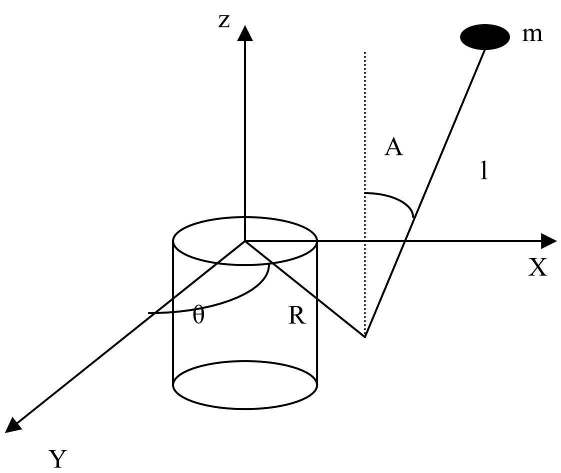

Figure-1. Simplified schematic picture of rotary inverted pendulum.

2.2. Derivation of Servo motor model

The transfer function of servo motor plant is developed with the block diagram as shown in Figure-2.

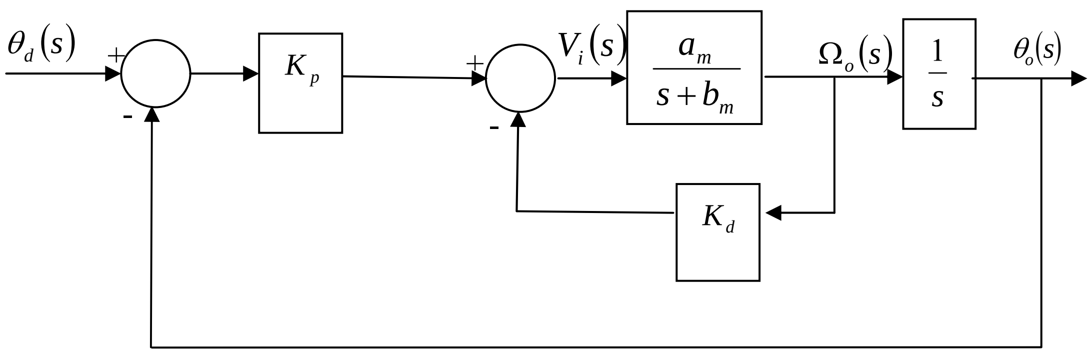

Figure-2. Block diagram of Servo motor.

$$\frac{\theta_0(S)}{v_i(S)} = \frac{\dfrac{\eta k_m k_g}{R_a J_{eq}}}{s\left(s + \dfrac{B_{eq}}{J_{eq}} + \dfrac{\eta k_m^2 k_g^2}{R_a J_{eq}}\right)} \quad \text{------------------------- (1)}$$

$$\frac{\theta_0(S)}{v_i(S)} = \frac{a_m}{s(s+b_m)} \quad \text{------------------------- (2)}$$

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Where

$$a_m = \frac{\eta k_m k_g}{R_a J_{eq}} \quad \text{and} \qquad \text{-------------------------- (3)}$$

$$b_m = \frac{B_{eq}}{J_{eq}} + \frac{\eta k_m^2 k_g^2}{R_a J_{eq}} \qquad \text{-------------------------- (4)}$$

Also from the block diagram, the output torque T_L , in s- domain is

$$T_L(s) = \frac{\eta k_m k_g}{R_a}\left[V_i(s) - k_m k_g \Omega_o(s)\right] \qquad \text{-------------------------- (5)}$$

Where Ω_o (s) is the angular velocity of arm in s- domain. $T_L(t) = K_1 V_i(t) - K_2 \omega(t)$ ------------------------- (6)

Where

$$K_1 = \frac{\eta k_m k_g}{R_a} \quad \text{and} \qquad \text{------------------------ (7)}$$

$$K_2 = \frac{\eta k_m^2 k_g^2}{R_a} \qquad \text{------------------------ (8)}$$

The parameters used for servo motor model [14] are defined in Table-1.

Table-1. Servo motor model parameters.

Symbol	Description	Value	Unit
R_a	Armature resistance	2.6	Ω
L_a	Armature inductance	negligible	
K_m	Motor voltage constant	0.00767	V-s/rad
K_τ	Motor torque constant	0.00767	N-m/A
J_m	Armature inertia	3.87×10^{-7}	Kg m^2
J_{tach}	Tachometer inertia	0.7×10^{-7}	Kg m^2
K_g	High gear ratio	(14) (5)	
B_{eq}	Equivalent viscous friction referred to the secondary gear	4×10^{-3}	Nm/(rad/s)
η_{mr}	Motor efficiency due to rotational loss	0.87	
η_{gb}	Gearbox efficiency	0.85	
J_L	Load inertia	5.2823×10^{-5}	Kg m^2
V_i	Motor input voltage	6	volts
J_{eq}	Equivalent inertia	0.0023	Kg m^2

2.3. Derivation of Inverted Pendulum model

To derive a dynamic system model, the coordinate frame systems shown in Figure-3 are introduced. With some standard assumptions such as zero friction, rigid objects etc, the dynamic model are given as follows:

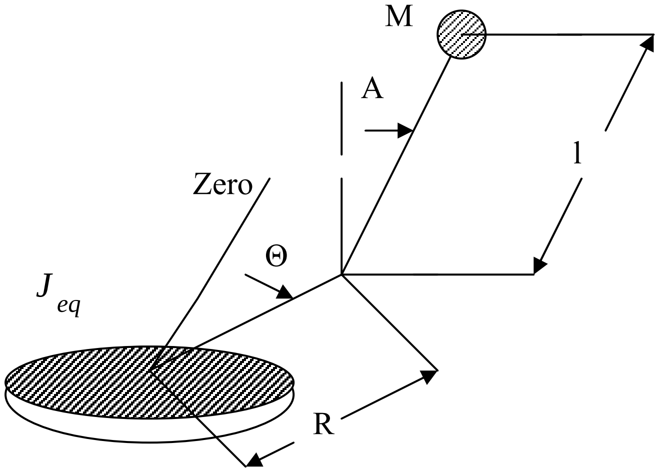

Figure-3. An illustrative configuration (swinging up).

$$\left(J_{eq}+mr^2\right)\dot{\omega}-\frac{1}{2}mLr\cos\alpha\,\dot{v}+\frac{1}{2}mLr\sin\alpha v^2+B_{eq}w=T_L \quad \text{------------- (9)}$$

$$\frac{1}{3}mL^2\dot{v}-\frac{1}{2}mLr\cos\alpha\,\dot{w}-\frac{1}{2}mgL\sin\alpha=0 \quad \text{------------ (10)}$$

The above equations describing the dynamics of the model are highly non linear.
The parameters of the pendulum are defined in Table-2.

Table-2. Pendulum parameters.

Symbol	**Description**	**Value**	**Unit**
m	mass of the pendulum	0.125	Kg
L	pendulum length	16.75	cm
r	length of the arm	21.5	cm
g	Gravitational acceleration	9.8	m/ s^2

Variable parameters

α	Pendulum angle
θ	Servo gear angular displacement
ω	Servo gear angular Velocity
v	Pendulum angular velocity

2.4. Linearized Inverted Pendulum model

A linear approximation to the non-linear system equations is obtained using the small angle formula.

$$\dot{x}(t)=Ax(t)+Bu(t) \quad \text{------------------- (11)}$$

$$y(t)=Cx(t) \quad \text{------------------- (12)}$$

Where,

$$x(t)=\left[\theta \quad \alpha \quad \omega \quad v\right]^T \quad \text{------------------- (13)}$$

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$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \dot{\omega} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 52.8058 & -22.9637 & 0 \\ 0 & 189.5215 & -44.2137 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \omega \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 40.7813 \\ 78.5192 \end{bmatrix} v_i(t) \qquad \text{---------------- (14)}$$

3. SWING UP USING A SIMPLE PD POSITIVE FEEDBACK CONTROLLER

As stated above, the goal of the controller is to swing up the pendulum from stable “down” position to the unstable equilibrium “up” position and be balanced there. The overall controller can actually be divided into 1) the swing up controller, 2) the balancing /stabilizing controller and 3) the catching controller/mode switching controller.

Many different control algorithms can be used to perform the swing up control such as, trajectory tracking, rectangular reference input swing up type, Pulse Width Modulation (PWM) etc. in a controlled manner that energy is gradually added to the system to bring the pendulum to the inverted position. Here, a positive feedback PD controller is proposed because of its simple structure, effectiveness and easy tuning. The block diagram representation of swing up controller is as shown in Figure-3.

Now we start to derive the PD control law for our SERIP system. Here a positive feedback loop is used to swing up the pendulum. It actually consists of two loops as shown in Figure-4. The outer loop specifies the trajectory for the arm angle and at the same time excites the internal dynamics to swing the pendulum to the balancing position. By moving the arm back and forth, one can eventually bring up the pendulum.

3.1. Outer loop PD controller design

It is fairly intuitive to design the outer loop as follows:

$$\theta_d = P\alpha + D\dot{\alpha} \qquad \text{--------------- (15)}$$

Where θ_d is the given trajectory of the arm and α is the pendulum angle deviated from the down hanging position which is positive in the clockwise direction and negative in the counter clockwise direction. Note that α is limited within $\pm$ 180° (wrapped around).

The values of the two parameters P and D play a key role in bringing up the pendulum smoothly. To prevent the pendulum from colliding with the other components, we need to limit θ within $\pm$ 90°. Initially, P can be chosen as 0.02. To properly choose D, a compromise should be made between increasing the reaction time and decreasing the noise amplification. In our system, D is set to be 0.015(sec.) at first. P and D can be tuned to adjust the “positive damping” in the system and meet the experiment criterions.

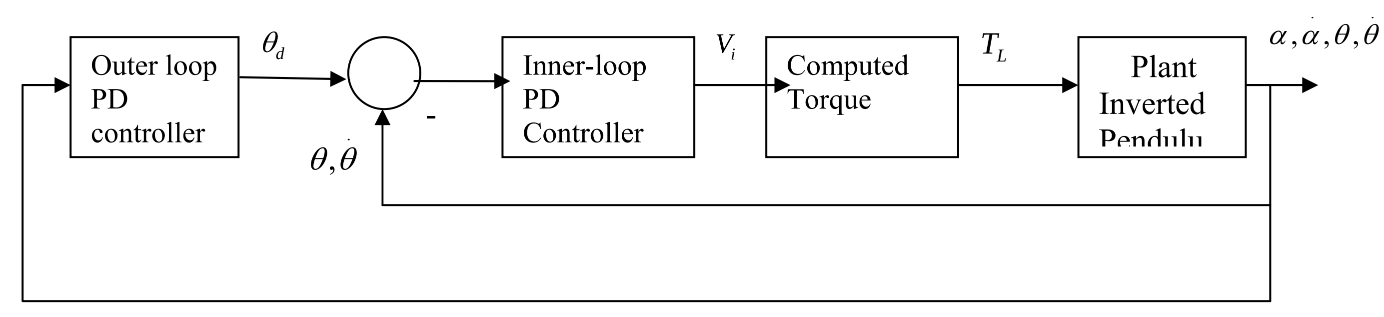

Figure-4. Block diagram representation of Swing up controller.

3.2. Inner loop PD controller design

The inner loop performs the position control of the arm. For the servo arm to track the desired position, a feedback PD controller is designed as follows:

$$V_i(s) = K_p[\theta_d(s) - \theta_o(s)] - K_d\Omega_o(s) \qquad \text{----- (16)}$$

Where K_p and K_d is the parameters to be tuned.

The first thing to do is to find out the closed-loop transfer function between the input and the output of the arm angle to satisfy the following performance requirements.
The percent overshoot should be less than 10%
The time to first peak should be i.e. t p = 0.115sec

Where θ_d is the desired position to track? By some mathematical manipulations, the closed loop transfer function can be expressed as

$$\frac{\theta_o(s)}{\theta_d(s)} = \frac{K_p a_m}{s^2 + (K_d a_m + b_m)s + K_p a_m} \quad \text{------- (17)}$$

According to performance requirements, a natural frequency, ω_n = 45.53 rad/sec and damping ratio ς =0.8 can be obtained. The closed – loop response of the arm could be considerably faster than that of the pendulum and a better compromise between overshoot and transient time can be achieved.

Comparing the coefficients of the closed loop system with a standard second-order characteristic polynomial,

$$s^2 + 2\varsigma\omega_n s + \omega_n^2 \quad \text{-------- (18)}$$

the controller gains are obtained as given below:

$$K_p = \frac{\omega_n^2}{a_m} = 99.0983 \quad \text{--------- (19)}$$

and

$$K_d = \frac{2\varsigma\omega_n - b_m}{a_m} = 2.8624 \quad \text{---------- (20)}$$

Where a_m and b_m are calculated as per equations (3) and (4).

4. BALANCE CONTROLLER

When the pendulum is almost upright, a stabilizing controller should be implemented to maintain it in the upright position and reject the possible external disturbance. Traditional methods employ state feed back control to stabilize, where pole placement and LQR are typical ones to be considered.

4.1. Conventional LQR controller design

The well known Linear Quadratic Regulator (LQR) is a cost-oriented optimal control method, where with certain weighting matrices a quadratic form cost

$$J = \int_0^\infty \left[x^T(t)Qx(t) + u^T(t)Ru(t)\right]dt \quad \text{------- (21)}$$

is minimized and the optimal state gains can be derived. (22) based on the linearized plant model.

With the linearized model of the system, the following parameters are assigned to design optimal gains by LQR method:

Open loop poles are found as 0, -27.2044, 10.9890, and 6.7483. Since one pole is on the right half of the s- plane the system is unstable.
The feed back gain matrix
K = [-316.2278 501.6609 -83.8125 47.8616]
of the controllers are assigned when Q = diag(1 0.5 1e-4 1e-4) and R = 1e-5 for the control law

$$u(t) = -Kx(t) \quad \text{---------- (22)}$$

On substituting (22) in to (11) yields

$$\dot{x}(t) = [A - BK]x(t) = A_f\, x(t) \quad \text{--------- (23)}$$

The closed loop system (A_f) poles are 1.0e+002 * (-2.6948, -0.8068, -0.0644 + 0.0326i, -0.0644 - 0.0326i).They all lie in the left half of s-plane and shows the closed loop system is stable.

4.2. Proposed fuzzy balance controller design

For the proposed fuzzy balance controller, alpha and alphadot are considered as two input variables and the servo motor voltage is the output variable. Each variable is decomposed into a set of fuzzy regions, which are called labels. The most popular labels are Negative Big (NB), Negative Small (NS), Zero (Z), Positive Small (PS) and Positive Big (PB). Based on experience and understanding of system characteristics, membership functions of the premise and consequent parts are defined as triangular membership functions. Equivalently, when α and $\dot{\alpha}$ are NB (PB), the voltage V_i need to be exerted such as PB (NB), so that the pendulum will rotate clockwise (counterclockwise) to the upright position, then α and $\dot{\alpha}$ will converge toward Z. The fuzzy inference rules are summarized in Table-3.

Table-3. Inference rules for fuzzy balance controller.

α \ α	NB	NS	Z	PS	PB
NB	PB	PB	PB	PS	Z
NS	PB	PB	PS	Z	NS
Z	PB	PS	Z	NS	NB
PS	PS	Z	NS	NB	NB
PB	Z	NS	NB	NB	NB

5. SWITCHING CONTROLLER

The switching/catching controller determines when to switch between two controllers (swing up controller and balancing controller). When the pendulum is at the upright position, it is easy to keep it up. However, keeping it upright nicely is not enough. It is required that

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the whole control process (swinging up, balancing and the mode switching) must be robust. That is, when any disturbance is applied to the pendulum, the controller can switch properly between the swing up control and the balance control. The pendulum can swing up, approach the upright position and switch to the balance control smoothly.

With both the swing up controller and balance controller complete, a transition algorithm is needed to connect them. The controllers are switched by a mere switching function. In the simulation, the control algorithm consists of the error between the current states and the goal states in the top configuration. Observing the output of the voltage the results of the simulation are very smooth and stable.

The switching function for the mode change over is done by checking the absolute values of alpha and alpha dot for less than 0.1rad and 0.2rad/sec respectively. If the condition is satisfied balance control will be selected otherwise the controller will remain in the swing up mode.

6. SIMULATION RESULTS

The simulation results of proposed control system for the inverted pendulum with the SIMULINK in MATLAB 7.0 are shown in Figures 5a,6a,7a, 8a and 9a and 5b,6b,7b,8b and 9b, respectively for conventional Hybrid controller and Intelligent Hybrid Controller.

It can be seen from Figure-5a that the pendulum can be swung up from the natural pendant position to upright position in about 4.95 seconds by PD controller. When the upright position has been reached, the system is switched to stabilizing control mode to stabilize the pendulum in its upright position. Though the change, swing up to stabilization, occurs almost simultaneously using the intelligent controller, in comparison with Figure-5a, in Figure-5b, an excellent stabilization controller response is obtained using intelligent controller. In figure 5a, we observe that the conventional system requires a greater number of swings (13) to stabilize than the proposed system does (10).

Figures 6a and 6b show the response of the angular velocity of the pendulum using conventional and the proposed intelligent controller. Here, again the response is better without a spike which appears in figure 6a at 5.5 seconds.

Figures 7a and 7b show the movement of the arm back and forth with respect to the movement of the pendulum both for conventional controller and intelligent controller respectively.

Figures 8a and 8b illustrates the angular velocity of the arm in both the modes.

The response shown in Figures 9a and 9b is the control signal of the proposed hybrid control system for swing up and stabilizing control using conventional and intelligent methods. It is seen that during the swing up state, the control signal is larger than the control signal in the stabilizing state. This is relevant because the large amount of energy is required to swing the pendulum up to its upright position and the small amount energy is only required to stabilize the pendulum.

Figures 10 and 11 explain the location of poles and the responses of various system parameters for initial conditions, respectively.

A. Simulated result for swing up control
(CONVENTIONAL CONTROLLER)

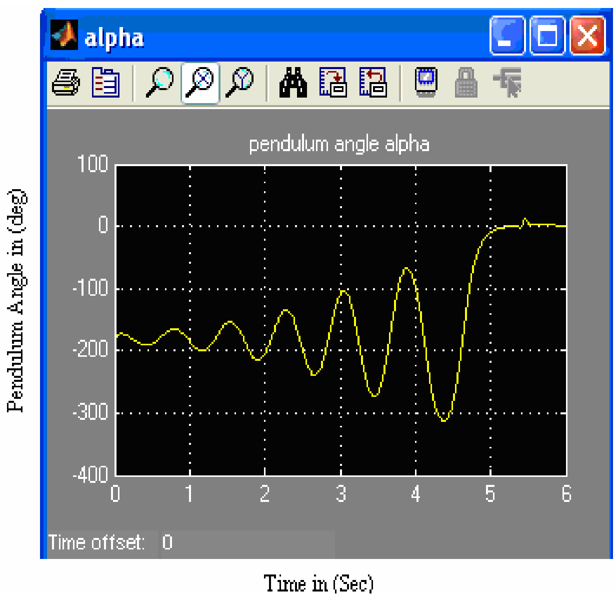

Figure-5a. Simulation result for Pendulum angle alpha (Conventional)

B. Simulated result for swing up control
(FUZZY CONTROLLER)

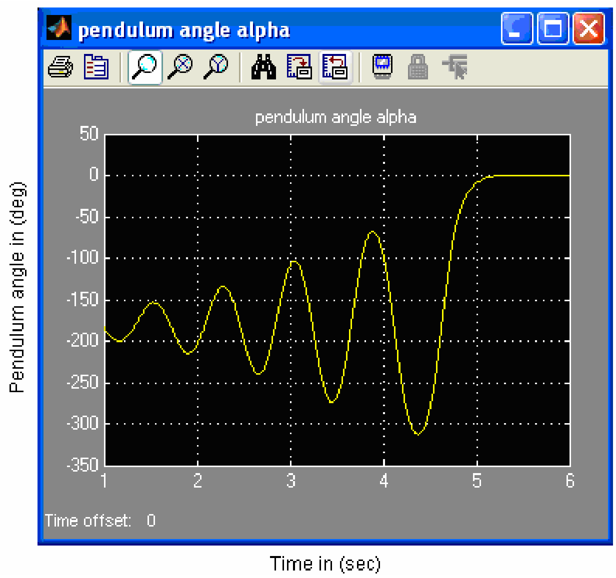

Figure-5b. Simulation result for Pendulum angle alpha (Fuzzy)

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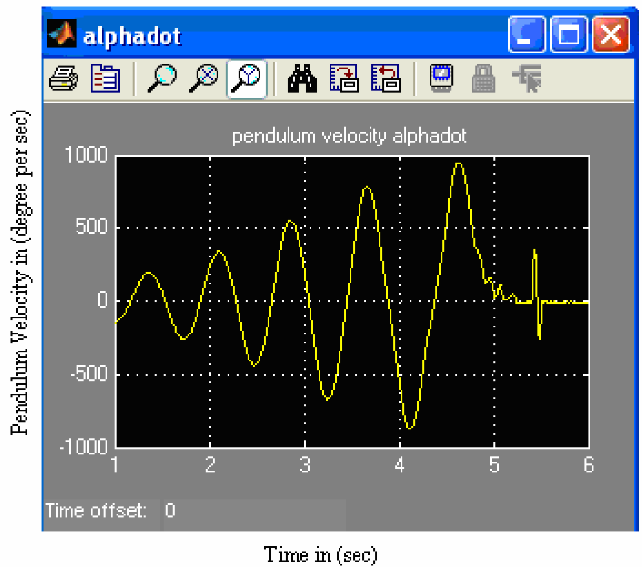

Figure-6a. Simulation result for Pendulum velocity alphadot (Conventional)

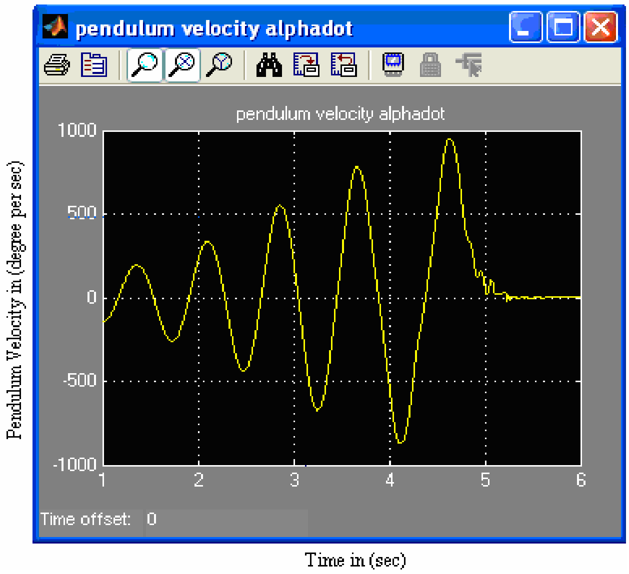

Figure-6b. Simulation result for Pendulum velocity Alphadot (Fuzzy)

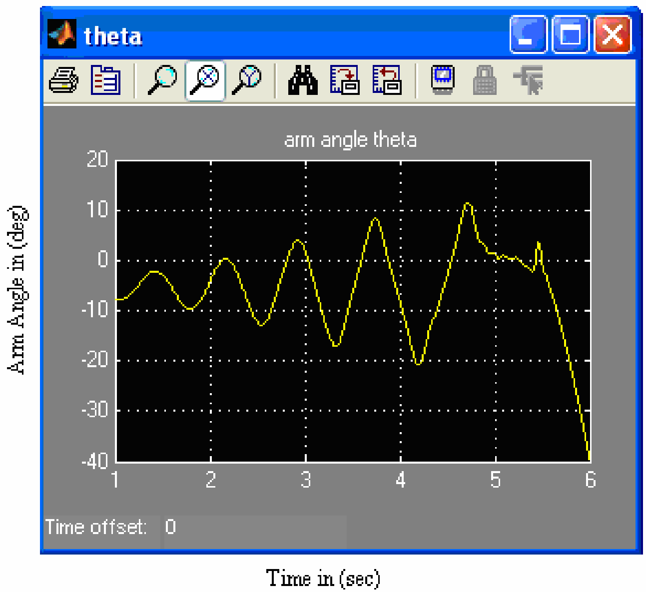

Figure-7a. Simulation result for arm angle theta (Conventional)

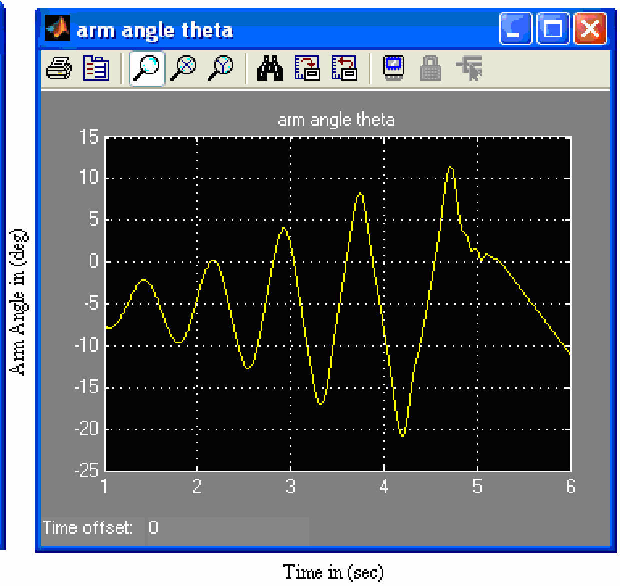

Figure-7b. Simulation result for arm angle theta (Fuzzy)

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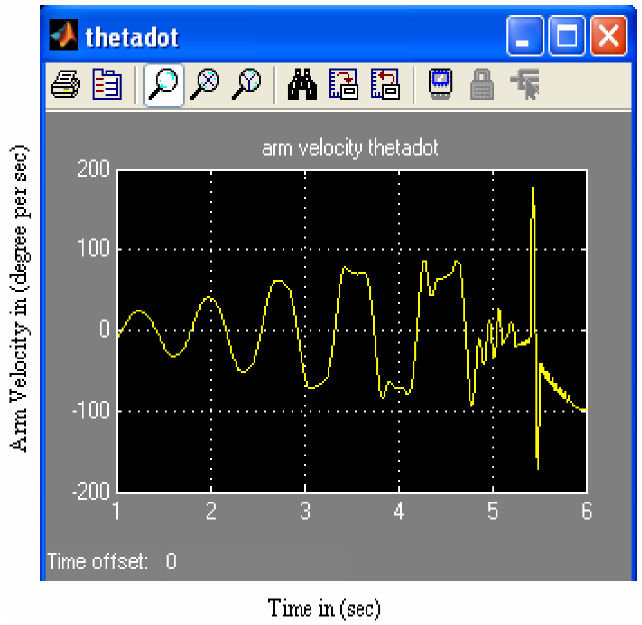

Figure-8a. Simulation result for arm velocity thetadot (Conventional)

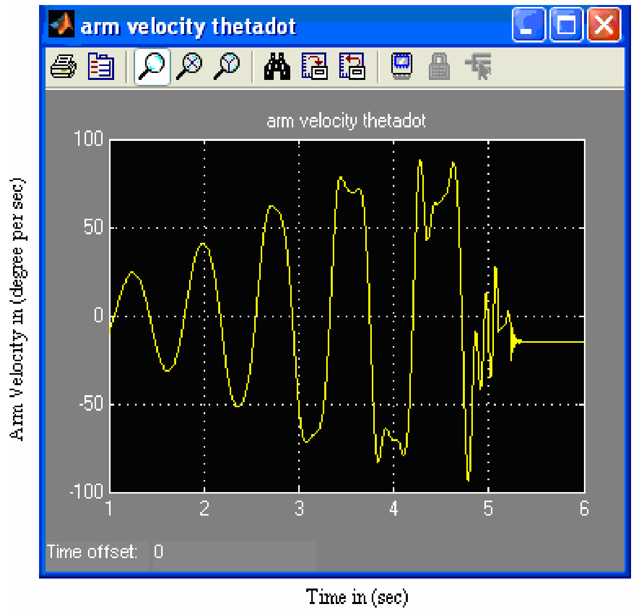

Figure-8b. Simulation result for arm velocity thetadot (Fuzzy)

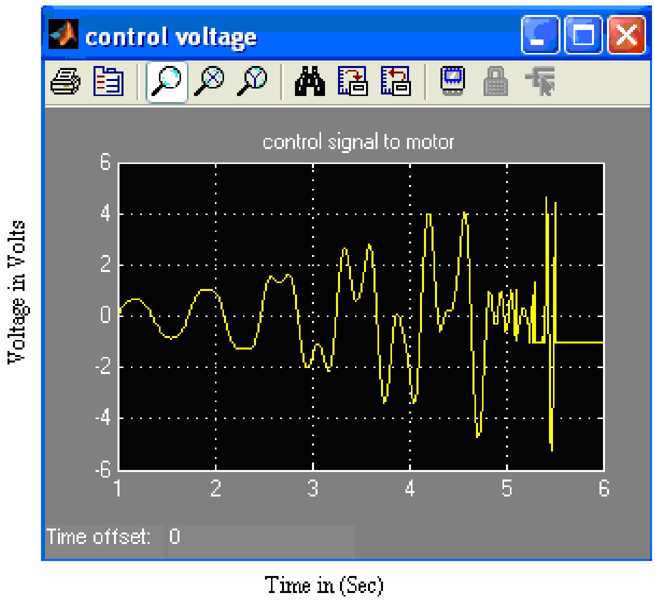

Figure-9a. Simulation result of voltage input to arm motor (Conventional)

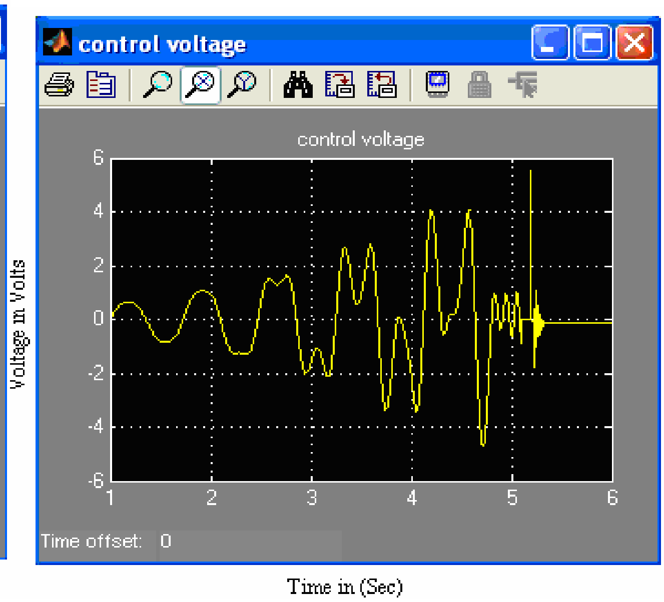

Figure-9b. Simulation result of voltage input to arm motor (Fuzzy)

C. Results for stabilization control

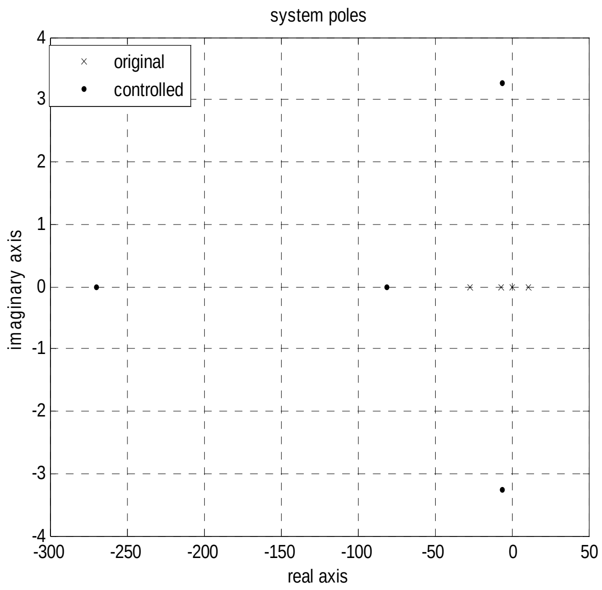

Figure-10. Location of open loop and closed loop Poles in the complex plane.

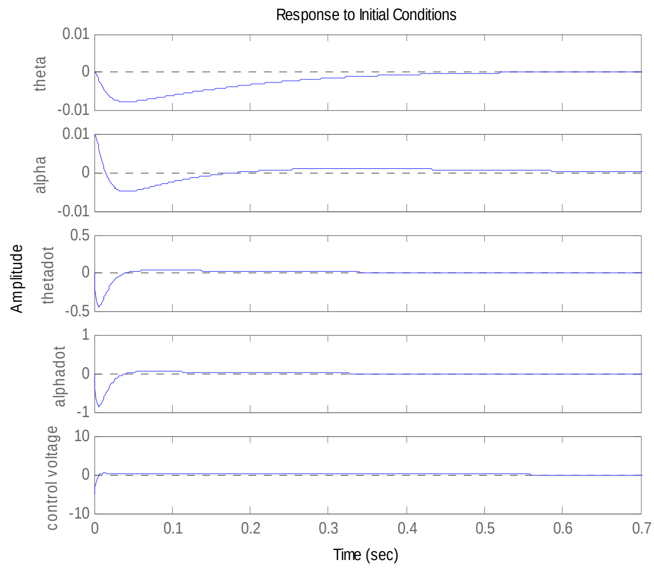

Figure-11. Response of various system parameters for initial conditions.

7. CONCLUSIONS

The Rotary Inverted Pendulum problem has been extensively studied in this paper. A hybrid controller for swinging up inverted pendulum pivoted on arm has been proposed in this paper. With the cooperative tasks of PD positive feed back position controller and proposed Fuzzy balance controller for stabilization, the control of inverted pendulum on arm has been simulated. Similarly, with the cooperative tasks of PD position controller and optimal servo state feedback controller, the control of inverted pendulum on arm has been simulated. In conclusion the proposed method is more effective. Furthermore, the proposed control method yields the desired system performance despite its simplicity in design.

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