VOL. 4, NO. 5, JULY 2009 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences
©2006-2009 Asian Research Publishing Network (ARPN). All rights reserved.
www.arpnjournals.com

THE EFFECTS OF TIME-MOMENTS AND MARKOV-PARAMETERS ON REDUCED-ORDER MODELING

S. K. Mittal[1], Dinesh Chandra[2] and Bharti Dwivedi[3]
[1]U. P. Technical University, Lucknow, India
[2]Department of Electrical Engineering, Motilal Nehru National Institute of Technology, Allahabad, India
[3]Department of Electrical Engineering, Institute of Engineering and Technology, Lucknow, India
E-Mail: mittalsai@rediffmail.com

ABSTRACT

This note presents a study of the effect of time-moments and Markov-parameters on reduced order modeling and to identify suitable combination of time-moments and Markov-parameters to highlight the significance of retaining or near retaining a few terms (time-moments/Markov-parameters) in excess of r terms in arriving at a good overall time response approximation, where r denotes the order of the reduced-order model. To identify appropriate combination of time-moments and Markov-parameters of the system to be retained in the reduced-order model for obtaining good overall time response approximation, system under consideration are of nature: Critically-damped system, under-damped system with small settling time and under-damped system with large settling time.

Keywords: model reduction, padé approximation, routh criterion.

INTRODUCTION

In general, various reduced-order modeling procedures do not indicate any set of criterion for determining the optimum order of the model which can approximate the system adequately. Reducing the order of high-order linear systems has been studied by several authors. Some of the reported methods require the computation of eigen-values and some others use certain optimization procedures. An attractive simplification procedure which requires neither optimization nor eigen-values calculation, involving simple algebraic calculations of finite number of steps is frequency-domain method, Padé approximation. In this method, the approximants are so selected that some consecutive coefficients of model power-series expansion and those of the system coincide. Consider a single-input-single-output system described by the transfer function:

$$G_n(s) = \frac{a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_n}{s^n + b_1 s^{n-1} + \ldots + b_n} \quad (1)$$

$$= t_1 + t_2 s + \ldots + t_n s^{n-1} + \ldots \quad (2)$$

(Expansion around $s = 0$)

$$= M_1 s^{-1} + M_2 s^{-2} + \ldots + M_n s^{-n} + \ldots \quad (3)$$

(Expansion around $s = \infty$)

The problem is to determine its stable reduced-order (rth-order) approximant:

$$G_r(s) = \frac{\hat{a}_1 s^{r-1} + \hat{a}_2 s^{r-2} + \ldots + \hat{a}_r}{s^r + \hat{b}_1 s^{r-1} + \ldots + \hat{b}_r} \quad (4)$$

$$= \hat{t}_1 + \hat{t}_2 s + \ldots + \hat{t}_r s^{r-1} + \ldots \quad (5)$$

$$= \hat{M}_1 s^{-1} + \hat{M}_2 s^{-2} + \ldots + \hat{M}_r s^{-r} + \ldots \quad (6)$$

The usefulness of techniques for deriving low-order approximations of high-order systems has already been accepted due to the advantages of reduced computational effort and increased understanding of the original system. Consequently, a large number of time-domain and frequency-domain system simplification techniques have been developed to suit different requirement. Amongst them, a frequency domain method is Padé approximation in which $2r$ terms of the power series expansion (time moments) of the high-order (nth-order) transfer function $G_n(s)$ are fully retained in low-order (rth-order) model $G_r(s)$. The Padé approximation does not guarantee the stability of the reduced-order model. To overcome the problem of stability, several stable reduction methods such as Routh approximation [1-3], the Hurwitz polynomial approximation [4], the stability equation method [5] and the method using Michailov stability criterion [6] have been proposed. The Routh approximation [1-3] has the drawback of matching only the first r time moments ($t_1, t_2, \ldots, t_r$) of $G_n(s)$ to the respective time moments ($\hat{t}_1, \hat{t}_2, \ldots, \hat{t}_r$) of $G_r(s)$ (in recent years the extension of Routh approximation techniques [1-3] to interval systems has attracted the attention of many researchers [7-11]. Later Shamash [12] considered the effect of including some Markov parameters ($M_1, M_2, \ldots$) along with time moments, which is generally essential to ensure both initial and steady state response approximation. However, the technique of [12] is again confined to matching of only r terms (α time moments and β Markov parameters, where $\alpha + \beta = r$). Several variants of Routh approximation were subsequently reported [13-16]; however, they again remain confined to only r terms matching for the purpose of preserving stability, a task which can be achieved arbitrarily [17,18]. Note that infinite numbers of stable models can be constructed if the objective is to match only r terms [18]. Thus, the basic problem is to match or near match a few terms in excess of r terms while preserving stability [19,

©2006-2009 Asian Research Publishing Network (ARPN). All rights reserved.

20]. Some attempt was made previously [21-23] to partially solve this problem. Singh [22] suggested a technique based on the successive variances of the model. The method [22] requires the determination of the stability region in terms of the free parameters. A modification of above technique was given by Lepeschy and Viaro [23]. Other closely related problems have also received attention [24-36]. Recently, geometric programming based (computer-oriented) methods [37, 38] for the solution of the Routh-Padé approximation problem are presented. In these methods [37, 38], first *r* time moments/Markov parameters are fully retained and the sum of the weighted squares of errors between a set of subsequent time moments/Markov parameters of the system and those of the model are minimized while preserving stability. These methods [37, 38] have the drawback that the question of finding some means (free of hit and trial) of deciding the values of the number of time moments/Markov parameters (say *m*) to be matched or near-matched and the weights to correspond to assured substantial improvement in system approximation as well as the question of establishing the existence of such values are left unresolved.

The reasons as to why, in certain cases, model obtained by considering only time-moments (i.e. not considering Markov-parameters) may turn out to be a good approximant are explained presently. Thus the basic problem is to match or near match a few terms in excess of r terms while preserving stability and to identify appropriate combination of time-moments and Markov-parameters of the system to be retained in the reduced-order model for obtaining good overall time response approximation. It is felt that the observations presented in this note will be helpful in formulating the optimization problem [37, 38].

EXAMPLES

Example l

Consider a fourth-order Critically-damped system given by Younseok Choo and Kim D. [43] with transfer function:

$$G(s)=\frac{2s^3+12s^2+18s+8}{s^4+6s^3+14s^2+16s+8} \qquad (7)$$

$$=1+0.25s-0.75s^2+0.5625s^3-0.125s^4-0.2031s^5-0.2976s^6+\cdots \qquad (8)$$

(Expansion around $s=0$)

$$=2s^{-1}+0s^{-2}-10s^{-3}+36s^{-4}-92s^{-5}+662s^{-6}-3180s^{-7}+\cdots \qquad (9)$$

(Expansion around $s=\infty$)

Routh approximants according to the technique of Pal J. [42] takes the form:

$$\hat{G}_3(s)=\frac{1.555s^2+3s+1.3333}{s^3+1.888s^2+2.6667s+1.3333}, \qquad (10)$$

$$=1.3333-0.41669s+0.1119s^2-0.6339s^3+1.42177s^4-2.0299s^5+2.56s^6+\cdots \qquad (11)$$

(Expansion around $s=0$)

$$=1.555s^{-1}+0.064155s^{-2}-2.93417s^{-3}+3.2977s^{-4}+1.5959s^{-5}-7.8961s^{-6}-6.2534s^{-7}+\cdots \qquad (12)$$

(Expansion around $s=\infty$)

The Padé approximants for third-order models can be obtained by considering α time-moments and β Markov-parameters: $\alpha+\beta=2r$

Third-order model by taking $\alpha=3$ and $\beta=3$ (equal number of time-moments and Markov-parameters) turns out to be:

$$\hat{G}_3(s)=\frac{2s^2+10.5s+7}{s^3+5.125s^2+8.5s+7}, \qquad (13)$$

$$=1+0.25s-0.75s^2+0.5625s^3-0.126s^4-0.179326s^5-0.23286s^6+\cdots \qquad (14)$$

(Expansion around $s=0$)

$$=2.3846s^{-1}-3.38566s^{-2}+9.1976s^{-3}-66.079s^{-4}+501.1695s^{-5}-3631.1824s^{-6}-25977.9057s^{-7}+\cdots \qquad (15)$$

(Expansion around $s=\infty$)

The Padé approximants for third-order model by considering only time-moments takes the form:
(With $\alpha=6$ and $\beta=0$)

$$\hat{G}_3(s)=\frac{2.2590s^2+1.89759s-0.02409}{s^3+1.765059s^2+1.90361s-0.02409}, \qquad (16)$$

Which is clearly unstable as the coefficients of the denominator are not of the same sign? The Padé approximants
(More Markov-parameters than time-moments with $\alpha=1$ and $\beta=5$), third-order model turns out to be:

$$\hat{G}_3(s)=\frac{2s^2+11.5s+13}{s^3+5.75s^2+11.5s+10.75}, \qquad (17)$$

$$=1.2093-0.2239s-0.2209s^2+1.4901s^3-0.0207s^4-1.93e-3s^5+0.1295s^6+\cdots \qquad (18)$$

(Expansion around $s=0$)

$$=2s^{-1}-0s^{-2}-10s^{-3}-34s^{-4}-80.5s^{-5}+71.875s^{-6}+146.97s^{-7}+\cdots \qquad (19)$$

(Expansion around $s=\infty$)

The Padé approximants (more time-moments than Markov-parameters with $\alpha=5$ and $\beta=1$,
Third-order model turns out to be:

$$\hat{G}_3(s)=\frac{2.3846s^2+17.8461s-13.53846}{s^3+8.9037s^2+14.461s-13.53486}, \qquad (20)$$

©2006-2009 Asian Research Publishing Network (ARPN). All rights reserved.

$$= 1 + 0.25s - 0.75s^2 + 0.5625s^3 - 0.126s^4 - 0.179326s^5 + 0.23286s^6 + \cdots \quad (21)$$

(Expansion around $s = 0$)

$$= 2.3846s^{-1} - 3.38566s^{-2} + 9.1976s^{-3} - 66.079s^{-4} + 501.1695s^{-5} - 3631.1824s^{-6} - 25977.9057s^{-7} + \cdots \quad (22)$$

(Expansion around $s = \infty$)

Comparison in terms of step and impulse responses of (10), (13), (17) and (20) with that of original system (7) has been shown in Figure-1 and Figure-2. It can be observed that step and impulse responses of (13) almost match with that of (7), while other models show deviation from (7). Model (10) is a poor approximant to the system (7) as it retains r time-moments only. Models (20) and (17) retain $2r$ terms, but these are not good approximants as the choice of number of time-moments and Markov-parameters to be retained in the model are not appropriate. It is to be noted that model (20) retains 5 time-moments and one Markov-parameter ($\alpha = 5, \beta = 1$), while model (17) retains 1 time-moment and 5 Markov-parameters ($\alpha = 1, \beta = 5$). The Padé approximants for third-order model by considering only time-moments is clearly unstable as the coefficients of the numerator and denominator are not of the same sign (16).

Table-1 lists errors of matching of time-moments and Markov-parameters of the system with those of the models. Here, system under consideration is critically-damped. Model (13) which retains equal number of time-moment and Markov-parameters ($\alpha = 3, \beta = 3$) is an improved approximant in comparison to models (10), (17) and (20).

Thus, better reduced-order model for this class of system (critically-damped) would be one in which not only first r terms are fully matched and the errors of matching of $(r+1)th$ and subsequent terms (Markov-parameters) are minimal but also priority is given to the matching or near-matching of equal number of Markov-parameters and time-moments.

Table-1.

	Eqn. (10)	**Eqn.** (13)	**Eqn.** (17)	**Eqn.** (20)
$(1-\hat{t}_1/t_1)^2$	0.0000	0.0000	0.0000	0.0438
$(1-\hat{t}_2/t_2)^2$	0.0000	5.9049	0.0000	3.5933
$(1-\hat{t}_3/t_3)^2$	0.0000	1.03477	0.0000	0.4980
$(1-\hat{t}_4/t_4)^2$	0.008782	0.66788	0.0000	2.7194
$(1-\hat{t}_5/t_5)^2$	6.2216	2.7635	0.0000	2.7194
$(1-\hat{t}_6/t_6)^2$	0.03124	0.3480	0.0137	0.9810
$(1-\hat{t}_7/t_7)^2$	1.7848	1.11289	3.1766	2.03675
$(1-\hat{M}_1/M_1)^2$	0.04950	0.0000	0.03697	0.0000
$(1-\hat{M}_3/M_3)^2$	0.49925	0.065823	3.6854	0.0000
$(1-\hat{M}_4/M_4)^2$	0.82518	0.07922	8.0402	19.360
$(1-\hat{M}_5/M_5)^2$	1.03499	0.2000	6.44749	0.015625
$(1-\hat{M}_6/M_6)^2$	1.03299	0.18378	42.0574	0.7946
$(1-\hat{M}_7/M_7)^2$	1.0039	0.4158	51.396	1.09456

VOL. 4, NO. 5, JULY 2009 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences
©2006-2009 Asian Research Publishing Network (ARPN). All rights reserved.

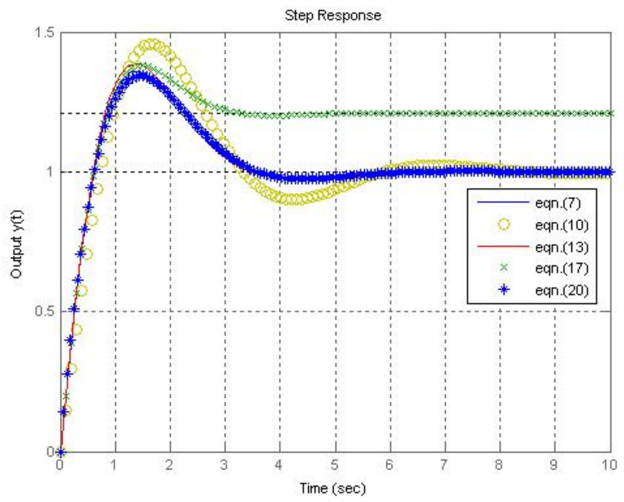

Figure-1. Step responses of original system (7) and its models.

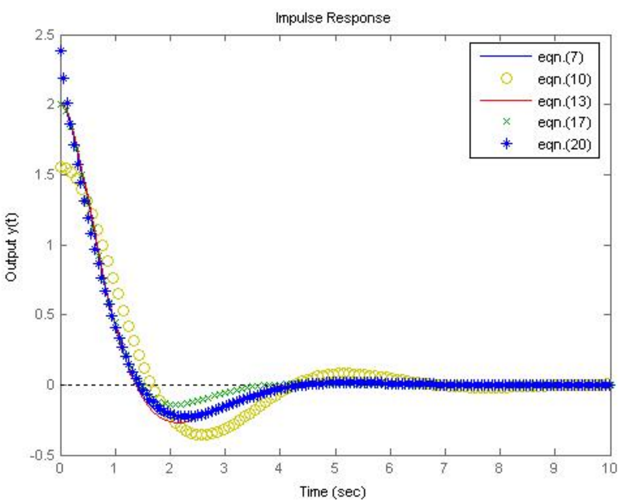

Figure-2. Impulse responses of original system (7) and its models.

Example 2

Consider the following eighth-order Under-damped system with small settling time system by Manigandan, Devarajan and Sivanandam [44] with transfer function:

$$G(s)=\frac{35s^7+1086s^6+13285s^5+82402s^4+278376s^3+511812s^2+482964s+194480}{s^8+21s^7+220s^6+1558s^5+7669s^4+24469s^3+46350s^2+45952s+17760} \quad (23)$$

$$=10.9505-1.391s+3.1872s^2-4.6863s^3+5.2881s^4-5.5638s^5+5.7008s^6+\cdots \quad (24)$$

(Expansion around $s=0$)

$$=35s^{-1}+351s^{-2}-1786s^{-3}-11842s^{-4}+104705s^{-5}+152601s^{-6}-3820956s^{-7}+\cdots \quad (25)$$

(Expansion around $s=\infty$)

The Padé approximants (more Markov-parameters than time-moments with $\alpha=1$ and $\beta=5$ third order model turn out to be:

$$\hat{G}_3(s)=\frac{35s^2+855.8962s+5328.9}{s^3+14.4256s^2+58.60150s+486.6364}, \quad (26)$$

$$=10.9505+0.4398s-0.3057s^2+0.0013s^3+0.0080s^4-3.7373e-4s^5 1.9486e-4s^6+\cdots \quad (27)$$

Expansion around $s=0$)

$$=35s^{-1}+351s^{-2}-1786s^{-3}-11842s^{-4}+104705s^{-5}+152601s^{-6}-382096s^{-7}+\cdots \quad (28)$$

(Expansion around $s=\infty$)

Routh approximants according to the technique of J. Pal [42] takes the form:

$$\hat{G}_3(s)=\frac{32.427s^2+37.8205s+22.2446}{s^3+2.7511s^2+3.6651s+2.0314}, \quad (29)$$

$$=10.9505-1.391s+3.1872s^2-9.5984s^3+13.5620s^4-13.0388s^5+9.8828s^6+\cdots \quad (30)$$

(Expansion around $s=0$)

$$=32.4257s^{-1}-51.3865s^{-2}+44.723s^{-3}-0.7074s^{-4}\ s=\infty)\ -57.7623s^{-5}+70.5542s^{-6}+19.0373s^{-7}+\cdots \quad (31)$$

(Expansion around $s=\infty$)

The Padé approximants (more Markov-parameters than time-moments with $\alpha=1$ and $\beta=5$ third order model turn out to be:

$$\hat{G}_3(s)=\frac{35s^2+855.8962s+5328.9}{s^3+14.4256s^2+58.60150s+486.6364}, \quad (32)$$

$$=10.9505+0.4398s-0.3057s^2+0.0013s^3+0.0080s^4-3.7373e-4s^5-1.9486e-4s^6+\cdots \quad (33)$$

(Expansion around $s=0$)

$$=35s^{-1}+351s^{-2}-1786s^{-3}-11842s^{-4}+52841s^{-6}-1136800s^{-7}+\cdots \quad (34)$$

(Expansion around $s=\infty$)

©2006-2009 Asian Research Publishing Network (ARPN). All rights reserved.

Table-2.

	$(1-\frac{\hat{t}_1}{t_1})^2$	$(1-\frac{\hat{t}_2}{t_2})^2$	$(1-\frac{\hat{t}_3}{t_3})^2$	$(1-\frac{\hat{t}_4}{t_4})^2$	$(1-\frac{\hat{M}_1}{M_1})^2$	$(1-\frac{\hat{M}_2}{M_2})^2$	$(1-\frac{\hat{M}_3}{M_3})^2$	$(1-\frac{\hat{M}_4}{M_4})^2$
Eqn.(29)	0.0000	0.0000	0.0000	1.0987	0.0000	1.9213	1.2010	1.0006
Eqn.(32)	0.0054	1.3142	1.0508	0.9999	0.0000	0.0000	0.0000	0.0000

Comparison in terms of step and impulse responses with that of original system (23) has been examined. Thus better reduced-order model for this class of system (under-damped with small settling time) would be one in which not only first r terms are fully matched and the errors of matching of ($r+1$) and subsequent terms (Markov-parameters) are minimal but also priority is given to the matching or near-matching of more number of Markov-parameters over time-moments.

CONCLUSIONS

It is observed that appropriate number of time-moments and/or Markov-parameters must be considered to ensure a good overall time response approximation. The main observations in this regard are as follows:

- In case of critically-damped systems, equal number of time-moments and Markov-parameters are to be retained.
- In case of under-damped systems with small settling time, more number of Markov-parameters is to be retained in the model than the time-moments.
- In case of under-damped systems with large settling-time, more number of time-moments is to be retained in the model than the Markov-parameters.

It is amply demonstrated that a stable reduced-order approximant based on fully retaining the first r terms (time-moments/Markov-parameters), some sort of optimization with regard to the (time-moments and/or Markov-parameters minimization of errors of matching of $(r+1)$th and subsequent terms) of the model with corresponding terms of the system must be inherently built-in the method and this feature will certainly produce better approximants.

ACKNOWLEDGMENT

The author wishes to thank N. L. Prajapati for his assistance.

REFERENCES

[1] M. F. Hutton and Friedland B. 1975. Routh approximations for reducing order of linear time-invariant systems. IEEE Trans. Automat. Contr. AC-20: 329- 337.

[2] Shamash Y. 1975. Model reduction usng the Routh stability criterion and the Pade approximation technique. Int. J. Control. 21: 475-484.

[3] A. S. Rao and S. S. Lamba and Rao S. V. 1978. Routh-approximant time domain reduced-order modeling for single-input single-output systems. IEE Proc. Control Theory Appl. 125: 1059-1063.

[4] Appiah. R. K. 1979. Pade methods of Hurwitz polynomial approximation with application to linear system reduction. Int. J. Control. 29: 39-48.

[5] T.C.Chen and C. Y. Chang and K. W. Han K. W. 1980. Stable reduced-order Pade approximants using stability equation method. Electron. Lett. 16: 345-346.

[6] Wan B. W. 1981. Linear model reduction using Michailov criterion and Pade approximation techniques. Int. J. Control. 3: 1073-1089.

[7] B. Bandyopadhyay and O. Ismail and Gorez R. 1994. Routh-Pade Approximation for Interval systems. IEEE Trans. Automat. Contr. 39(12):

[8] B. Bandyopadhyay and A. Upadhye and Ismail O. 1997. Γ-δ Routh approximation for interval systems. IEEE Trans. Automat. Contr. 42(12):

[9] C. Hwang and Yang S.F. 1999. Comments on the computation of interval Routh approximants. IEEE Trans. Autom. Control. 44: 1782-1787.

[10] G. V. K. R Sastry and G. R. Rao and Rao P.M. 2000. Large scale interval system modelling using Routh approximants. Electron. Lett. 36: 768-769.

[11] Y. Dolgin. And Zeheb E. 2003. On Routh-Pade model reduction of interval systems. IEEE Trans. Autom. Control. 48: 1610-1612.

[12] Shamash Y. 1980. Stable biased reduced-order models using the Routh method of Reduction. Int. J. Syst. Sci. 11: 641-654.

[13] V. Krishnamurthy and V.Sheshadri. 1978. Model reduction using Routh Stability criterion. IEEE Trans. Autom. Control. 23: 729-731.

[14] Pal J. 1983. Improved Pade approximants using stability equation method. Electron. Lett. 19: 26-427.

©2006-2009 Asian Research Publishing Network (ARPN). All rights reserved.

[15] Y. Bistritz and Shaked U. 1981. Stable linear systems simplification via Pade approximations to Hurwitz polynomials. Trans. ASME J. Dyn. Syst. Meas.Control. 103: 279-284.

[16] Shamash Y. 1981. 1981. Truncation method of reduction: a viable alternative. Electron. Lett. 17: 97-98.

[17] Singh V. 1979. Nonuniqueness of model reduction using the Routh approach. IEEE Trans. Autom. Control. 24: 650-651.

[18] Singh V. 1981. Remarks on system simplification using the Routh stability array. Proc. IEEE. 69: 662, 1981.

[19] N. Ashoor and Singh V. 1982. A note on low order modeling. IEEE Trans. Automat. Contr. 27: 1124-1126.

[20] N. Ashoor and Singh V. 1982. Remarks on system simplification under consideration of time response. Electron. Lett. 18: 496-497.

[21] Singh V. 1981. Improved stable Approximants using the Routh array. IEEE Trans. Automat. Contr. AC-26: 581-582.

[22] Singh V. 1981. Stable approximants for stable systems: A new approach. Proc. IEEE. 69: 1155-1156.

[23] A. Lepschy and Viaro U. An improvement in the Routh-Pade approximation techniques. Int. J. Control. 36: 643-661.

[24] Lucas T. N. 1988. Scaled impulse energy approximation for model reduction. IEEE Trans.Autom-at. Contr. 33: 791-793.

[25] V Puri, and Lam D. P. 1988. Stable model reduction by impulse response error minimization using Michailov criterion and Pade´ approximation. Trans. ASME J. Dyn. Syst. Meas.Control. 110: 389-394.

[26] C. S Hsieh and Hwang C. 1989. Model reduction of continuous-time system using a modified Routh-approximation method. IEE Proc. D. Control Theory Appl. 136: 151-156.

[27] Y. C Lee and C. Hwang and Hwang J. H. 1994. Model-reduction of SISO systems by Routh expansion and balancing method. J. Franklin Inst. 331B: 367-380.

[28] C. Hwang and J. H. Hwang, and Guo T. Y. 1995. Multifrequency Routh Approximants for linear systems. IEE Proc. Control Theory Applicat. 142: 351-358.

[29] Choo Y. 1999. Improvement to modified Routh approximation method. Electron. Lett. 35: 606- 607.

[30] Choo Y. 1999. Improvement to modified Routh approximation method (correction). Electron. Lett.1119.

[31] Choo Y. 1999. Direct method for obtaining modified Routh approximants. Electron. Lett. 35: 627-1628.

[32] Choo Y. 2001. Improved bilinear Routh approximation method for discrete time systems. Trans. SME J. Dyn. Syst. Meas. Control. 123: 125-127.

[33] Choo Y. 2002. Equivalence of bilinear Routh and Schwarz approximation methods for discrete- time systems. Electron. Lett.38: 761-762.

[34] C. Y. Hwang and Lee Y. C. 1997. A new family of Routh approximants. Circuits Syst. Signal Process. 16: 1-25.

[35] Lucas T. N. 1997. Constrained optimal Pade model reduction. ASME J .Dyna. Syst. Meas. Control. 119: 685-690.

[36] T. N. Lucas T. N. 2002. The bilinear method: a new stability-preserving order reduction approach. Proc. Inst. Mech. Engg. I. J. Syst. Contr. Engg. 216: 429-436.

[37] V. Singh and Dinesh Chandra and Kar H. 2004. Improved Routh-Pade approximants: A computer-aided approach. IEEE Trans. Automat. Contr.49: 292-295.

[38] Singh V. 2005. Obtaining Routh-Pade approximants using Luss-Jaakola algorithm. IEE Proc. Part I: 152.129-132.

[39] R. Luss and Jaakola T. 1973. Direct search and systematic reduction of size of search region. AICHE J. 19:760-766.

[40] P. C. Parks 1962. A new proof of the Routh-Hurwitz criterion using the second method of Lyapunov Proc. Camb. Philos. Soc. 694-702.

[41] K. Deb. 2002. Multi-objective Optimization using Evolutionary Algorithm. New York, John Wiley and Sons Ltd.

[42] Pal J. 1979. State reduced-order Padé approximants using Routh-Hurwitz array. Electron. Lett. 15. 25-26.

[43] Choo Y. and Dongmin K.2006. SISO Continuous System Reduction via impulse response Gramian by

©2006-2009 Asian Research Publishing Network (ARPN). All rights reserved.

iterative formulae. Trans. ASME J. Dyn. Syst. Meas. Contr. 128: 391-393.

[44] Manigandan T., Devarajan N. and Svanandam S. N. 2005. Design of PID controller using reduced order model. Academic Open Internet Journal. 15: 1-15.