



MULTI-RESPONSE OPTIMIZATION USING ANOVA AND DESIRABILITY FUNCTION ANALYSIS: A CASE STUDY IN END MILLING OF INCONEL ALLOY

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ABSTRACT

Nickel-based super alloys are classified as 'difficult to machine' materials due to its inherent characteristics such as high hardness, and toughness, high strength at elevated temperatures, low thermal conductivity, ability to react with cutting inserts, and ability to weld onto the surface of the cutting insert. The present study investigated the parameter optimization of end milling operation for Inconel 718 super alloy with multi-response criteria based on the Taguchi method and desirability function analysis. Experimental tests were carried out based on an L9 orthogonal array of Taguchi method. The influence of machining factors cutting speed, feed rate and depth of cut were analyzed on the performances of surface roughness and material removal rate. The optimum cutting conditions are obtained by Taguchi method and desirability function. The analysis of variance (ANOVA) is also applied to investigate the effect of influential parameters. A regression model was developed for surface roughness and material removal rate as a function of cutting velocity, feed rate and depth of cut. Finally, the confirmation experiment was conducted for the optimal machining parameters, and the betterment has been proved.

Keywords: end milling, inconel super alloy, machinability, Taguchi method, ANOVA, multi-response optimization, desirability function analysis

INTRODUCTION

Nickel-based super alloys finds wider applications in modern industries such as space vehicles, rocket engines, experimental aircrafts, nuclear reactors, submarines, steam power plants, gas turbines, nuclear reactors, petrochemical equipments and other high-temperature applications [1, 2]. Among the nickel-based super alloys, Inconel 718 is the most frequently used. However due to its properties such as high tensile strength, abrasiveness, work hardening, high hardness, low thermal conductivity, strong tendency to weld and formation of built-up edge, it is difficult to machine Inconel 718 [3, 4]. They are known to be among the most difficult-to-cut materials. Several researchers have studied the effect of cutting conditions in machining of nickel based super alloys [5, 6, 7, 8, 9]. Most of the research on machining Inconel alloy is concentrated mainly on the study of cutting tool wear and wear mechanism [10, 11]. Poor selection of machining parameters causes cutting tools to wear and break quickly as well as economical losses such as damaged work-piece and poor surface quality [12, 13, 14].

The manufacture of aerospace components involves a variety of machining operations such as turning, facing, milling, and drilling. Among various machining processes, the end-milling process is one of the most widely used material removal processes in manufacturing industry for finish milling of this advanced material. The cutting operations by the end mills can be as simple as a face milling on the top of a flat surface with a rigid cutter or a milling of very complex parts [15]. Considerable research has been done in turning and milling of Inconel 718 using coated and uncoated carbide cutting tools [16]. Many researchers are optimizing various machining

process for single response criteria. But such types of studies were mainly focused on improving single-quality characteristics at a time. Few researchers have tried for modelling and optimization of multiple quality characteristics in the end milling of Inconel 718 alloys [12].

Further, most published literature have been concerned with the optimization of a single performance (or response) characteristic. But the performance of a machining process often characterized by a group of responses. If more than one response comes into consideration it is very difficult to select the optimal setting which can achieve all quality requirements simultaneously. Otherwise optimizing one quality feature may lead severe quality loss to other quality characteristics which may not be accepted by the customers. Handling the more demanding multiple performance characteristics are seldom considered in the literature. In order to tackle such a multi-response optimization problem, the present study is to find out the optimal setting of machining parameters on end milling of Inconel 718 super alloy. Parameter selection for end milling process is choosing the right combination of cutting speed, feed, and depth of cut to achieve desired surface finish with maximum material removal rate. Taguchi design approach is utilized for experimental planning during end milling of Inconel alloy. Based on the experiments conducted a mathematical model is developed for surface roughness and material removal rate using desirability function analysis. It is an attractive method for industry for optimization of multiple quality characteristic problems. The method makes use of an objective function, $D(X)$, called the desirability function and transforms an estimated response into a scale free value (d_i) called composite desirability [17, 18, 19].



Confirmation tests were performed by using experiments. ANOVA is performed to investigate the more influencing parameters on the multiple performance characteristics.

EXPERIMENTAL DETAILS

Materials and processes

The experimental study was carried out in wet cutting conditions on a Hass-US five-axis, high-speed CNC milling machine equipped with a maximum spindle speed of 12000 rpm, feed rate of 10 m/min and a 25-kW drive motor. CNC part programs for tool paths were created. The workpiece material used was Inconel 718 in the form of a 300mm (length) _ 52mm (width) _ 6mm (height) machine table to provide maximum rigidity. The workpiece material is mounted onto the machine table to provide maximum rigidity. The experimental setup of the workpiece for end mill is shown in Figure-1. The detailed information on chemical composition and mechanical properties of this Inconel 718 alloy is provided in Tables 1 and 2, respectively. The tool used for performing end milling operation is uncoated tungsten carbide (10mm diameter, 4 -flutes) produced by Sandvik. Tools with four teeth are selected for better surface quality. The same tool was used until maximum flank wear reached VB_{max} 0.2 mm. The machined surface was measured at three different positions using a surf test (Make - Mahr surf test) measuring instrument with the cutoff length 2.5 mm and the average surface roughness (R_a) value is recorded in microns. Material removal rate (MRR) is used as another performance measure to evaluate a machining performance. Material removal rate is expressed as the amount of material removed under a period of machining time and is expressed in mm^3/sec .



Figure-1(a). Experimental set up for end milling operation.



Figure-1(b). Set up for surface roughness measurement.

Table-1. Chemical composition of Inconel 718 alloy (wt %).

Elements	C	Mn	Si	Ti	Al	Co	Mb	Cb	Fe	Cr	Ni
Percentage	0.08	0.35	0.35	0.6	0.8	1.0	3.0	5.0	17.0	19.0	52.82

Table-2. Mechanical properties of Inconel 718 alloy.

Ultimate strength (MPa)	Yield point (MPa)	Elongation (%)	Hardness (HRC)
1260 – 1390	1041 – 1160	14 - 19	40 - 45

Plan of experiments

In recent years, the Taguchi method has become a powerful tool for improving productivity during research and development so that high quality products can be produced quickly and at low cost. Taguchi's parameter design is an important tool for robust design. Taguchi method uses a special design of orthogonal arrays to study the entire parameter space with a small number of experiments [20, 21, 12, 22].

Table-3. Parameters and their levels.

Parameter,	Units	Levels		
		1	2	3
Cutting speed (v)	m/min	25	50	75
Feed rate (f)	mm/tooth	0.06	0.09	0.12
Depth of cut (d)	mm	0.2	0.4	0.6



The methodology of Taguchi for three factors at three levels is used for the implementation of the plan of experiments. The degrees of freedom required for the study is six and Taguchi's L9 orthogonal array is used to define the 9 trial conditions. Only the main effects are of interest and factor interactions are not studied. The process parameters and levels are listed in Table-3. Each of the 9 trials or process designs is replicated twice and the average response values are used for the analysis. Table-4 shows the experimental layout and corresponding average test results.

DETERMINATION OF OPTIMAL MACHINING PARAMETERS

Desirability function analysis (DFA)

Derringer and Suich [21] popularized the concept of desirability function analysis as a simultaneous optimization technique which proved to be useful in solving multi-response problems. This method considers an objective function initially which transforms the existing values in to a scale free value called desirability. Later composite desirability is evaluated based on which the optimum level of parameters is decided to satisfy minimized surface roughness and maximized material removal rate. The steps involved in the optimization process are detailed below.

Step-1: The first step involves the calculation of desirability index (d_i) for each of the factors viz., surface roughness and material removal rate. The desirability index values calculated are listed in Table-5. It is calculated based on the desirability piece wise function which is shown in equation (1), equation (2) and equation (3), respectively for the cases of nominal the best, smaller the better and larger the better. In this study surface roughness is to be minimized and material removal rate is to be maximized, hence desirability index values for surface roughness and material removal rate are calculated based on the equation (2) and equation (3), respectively.

Step-2: The second step is to evaluate the composite desirability based on the equation (4).

Step-3: The third step is to determine the optimum combination of levels of parameters based on the highest composite desirability value. Also the effect of parameters on the responses considered is estimated.

$$d_i = \begin{cases} \left(\frac{\hat{y} - y_{\min}}{T - y_{\min}} \right)^s, & y_{\min} \leq y \leq T, \quad s \geq 0 \\ \left(\frac{\hat{y} - y_{\max}}{T - y_{\max}} \right)^t, & T \leq \hat{y} \leq y_{\max}, \quad t \geq 0 \\ 0, & \end{cases} \quad (1)$$

$$d_i = \begin{cases} 1, \\ \left(\frac{\hat{y} - y_{\max}}{y_{\min} - y_{\max}} \right)^r, & y_{\min} \leq y \leq y_{\max}, \quad r \geq 0 \\ 0, & \end{cases} \quad (2)$$

$$d_i = \begin{cases} 0, & \hat{y} \leq y_{\min} \\ \left(\frac{\hat{y} - y_{\min}}{y_{\max} - y_{\min}} \right)^r, & y_{\min} \leq \hat{y} \leq y_{\max}, \quad r \geq 0 \\ 1, & \hat{y} \geq y_{\max} \end{cases} \quad (3)$$

Where y_{\min} represents the lower tolerance limit of \hat{y} and y_{\max} represents the upper tolerance limit and 'r' represents weights considered. The s, t and r in Equations (1), (2), and (3) indicate the weights and are defined according to the requirement of the user.

Step-4: The next step is to perform ANOVA to observe the significance of each of the parameters in influencing the combined objective.

Step-5: The last stage is to calculate the predicted value of the response based on the optimum level of parameters obtained and to validate the results.

$$d_G = (d_1^{w_1} d_2^{w_2} \dots d_n^{w_n})^{\frac{1}{W}} \quad (4)$$

Table-5 shows the individual and composite desirability values for each of the experiment in L9 orthogonal array. Higher the composite desirability value better is the product quality. Therefore, on the basis of composite desirability, the factor effect can be estimated and the optimal level for each controllable factor can also be determined. The mean of the composite desirability for each level of the parameter is summarized and shown in Table-6. In addition, the total mean of the composite desirability for the 9 experiments is also calculated and listed in Table-6. Figure-2 shows the main effects plot for the composite desirability for the different levels of the processing parameters. Basically, the larger the composite desirability, the better is the multiple performance characteristics. However, the relative importance among the parameters for the multiple performance characteristics will still need to be known so that the optimal combinations of the process parameter levels can be determined more accurately [18].

ANALYSIS OF VARIANCE FOR COMPOSITE DESIRABILITY

The purpose of the analysis of variance is to investigate which machining parameters significantly affect the performance characteristic. This is accomplished by separating the total variability of the composite desirability, which is measured by the sum of the squared deviations from the total mean of the composite desirability, into contributions by each machining parameter and the error [17, 22]. First, the total sum of the



squared deviations SS_T from the total mean of the composite desirability γ_m can be calculated as (5):

Table-4. Experimental layout using an L9 orthogonal array and corresponding results.

Ex. No.	Process parameter			Average response values	
	Cutting velocity	Feed rate	Depth of cut	Surface roughness (microns)	Material removal rate (mm^3/sec)
1	1	1	1	0.21	4.308
2	1	2	2	0.25	4.480
3	1	3	3	0.29	4.503
4	2	1	2	0.2	5.643
5	2	2	3	0.27	5.731
6	2	3	1	0.27	5.904
7	3	1	3	0.21	6.906
8	3	2	1	0.23	7.080
9	3	3	2	0.27	7.530

Where p is the number of experiments in the orthogonal array and γ_j is the mean composite desirability for the j th experiment.

Table-5. Evaluated results of desirability function.

Exp. No.	Normalized values		Individual desirability after weighted		Composite desirability	
	Surface roughness	Material removal rate	Surface roughness	Material removal rate	Composite desirability	Rank
1	0.8889	0.0000	0.9428	0.0000	0.0000	8
2	0.4444	0.0534	0.6667	0.2311	0.1540	7
3	0.0000	0.0604	0.0000	0.2459	0.0000	8
4	1.0000	0.4142	1.0000	0.6436	0.6436	3
5	0.2222	0.4415	0.4714	0.6645	0.3132	6
6	0.2222	0.4952	0.4714	0.7037	0.3317	5
7	0.8889	0.8062	0.9428	0.8979	0.8465	1
8	0.6667	0.8604	0.8165	0.9276	0.7574	2
9	0.2222	1.0000	0.4714	1.0000	0.4714	4

The total sum of the squared deviations SS_T is decomposed in to two sources: the sum of the squared deviations SS_d due to each machining parameter and the sum of the squared error SSE . The percentage contribution of each of the machining parameter in the total sum of the

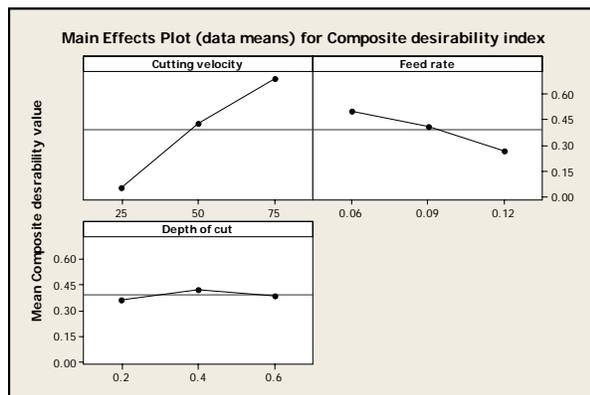
squared deviations SS_T can be used to evaluate the importance of the machining parameter change on the performance characteristic. In addition, the Fisher's F- test can also be used to determine which machining parameters have a significant effect on the performance characteristic.

**Table-6.** Response table for the composite desirability.

Process parameter	Average composite desirability				
	Level 1	Level 2	Level 3	Max-Min	Rank
Cutting velocity	0.0514	0.4295	0.6918*	0.6404	1
Feed rate	0.4967*	0.4082	0.2677	0.2289	2
Depth of cut	0.3630	0.4230*	0.3866	0.0599	3
Total mean value of the composite desirability = 0.3909					
* Optimum levels					

Table-7. Results of the analysis of variance.

Source of variation	DoF	SS	MS	F ratio	% C
Cutting velocity	2	0.62192	0.31096	8.17736	79%
Feed rate	2	0.08001	0.04000	1.05197	10%
Depth of cut	2	0.00548	0.00274	0.07204	1%
Error	2	0.07605	0.03803		10%
Total	8	0.78346			100%

**Figure-2.** Main effects plot for composite desirability index.

Usually, the change of the machining parameters has a significant effect on performance characteristic when F is large. Table-7 shows the results of ANOVA analysis. Results of analysis of variance indicate that cutting velocity is the most significant machining parameter followed by feed rate affecting the multiple performance characteristics.

DEVELOPMENT OF REGRESSION MODELS

Empirical models for material removal rate and surface roughness were developed using regression analysis. The correctness and the acceptability of the model are checked by the correlation coefficients R^2 and R^2 (adj). The equations were developed by using the Minitab 14 software and are as shown in equation (6) and equation (7), respectively for surface roughness and material removal rate.

Surface Roughness, in microns = $0.133 - 0.000267$ Cutting Velocity, m/min + 1.17 Feed Rate, mm/rev + 0.05 Depth of cut, mm (6)
For the surface roughness model, $R\text{-Sq} = 95.3\%$
 $R\text{-Sq}(\text{adj}) = 92.5\%$

Material Removal Rate, $\text{mm}^3/\text{sec} = 2.56 + 0.0548$ Cutting Velocity, m/min + 6.00 Feed Rate, mm/rev - 0.128 Depth of cut, mm (7)
For the material removal rate model, $R\text{-Sq} = 99.4\%$
 $R\text{-Sq}(\text{adj}) = 99.0\%$

PREDICTION OF OPTIMUM LEVELS

Once the optimal level of machining parameters is selected the final step is to predict and verify the improvement of the performance characteristics using the optimal level of the machining parameters. The estimated composite desirability $\hat{\gamma}$ using the optimum level of the machining parameters can be calculated as:

$$\hat{\gamma} = \gamma_m + \sum_{i=1}^q (\bar{\gamma}_i - \gamma_m) \quad (8)$$

Where γ_m is the total mean of the composite desirability, $\bar{\gamma}_j$ is the mean of the composite desirability at the optimum level and q is the number of machining parameters that significantly affects the multiple performance characteristics. Based on Equation (8) the estimated composite desirability using the optimal machining parameters can then be obtained. Table-8 shows the results of the confirmation experiment using the optimal machining parameters. The surface roughness (R_a) is improved from 0.21 to $0.19 \mu\text{m}$ and the material removal



rate (MRR) is greatly increased from 4.308 to 7.100 mm³/sec. It is clearly shown that multiple performance

characteristics in the end milling of Inconel 718 are greatly improved through this study.

Table-8. Comparison of predicted results using confirmation experiment.

	Initial machining parameters	Optimal machining parameters	
		Prediction	Experiment
Setting level	A ₁ B ₁ C ₁	A ₃ B ₁ C ₂	A ₃ B ₁ C ₂
Surface roughness (Ra)	0.21		0.19
Material removal rate (MRR)	4.308		7.21
Composite desirability value (DI)		0.8297	0.5141
Improvement in desirability value = 0.3156			

CONCLUSIONS

This paper addressed the multi-response optimization of machining parameters of Inconel 718 alloy in end milling. It has been established that desirability function analysis embedded in Taguchi analysis is an effective optimization tool for optimizing multi-response optimization problems. It was found that the optimal cutting parameters for this end milling process lies at 75m/min for cutting velocity, 0.06 mm/tooth for feed rate and 0.4 mm for depth of cut. Further significant improvement in machinability is observed and measured that there is a 64.8% increase in material removal rate and at the same time a 9.52% decrease in surface roughness. This encourages applying the desirability function for optimizing multi response problems with incomplete data. Analysis of variance shows that the cutting velocity is the most significant machining parameter followed by feed rate affecting the multiple performance characteristics with 56.88% and 34.64% influence, respectively.

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