TRANSIENT ANALYSIS OF SMART MATERIAL PLATES USING HIGHER ORDER THEORY

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ABSTRACT

Smart materials have many properties which are quickly responded to external stimuli such as temperature, moisture, stress, magnetic or electric fields. In this work an analytical procedure has been developed for investigating transient characteristics of smart material plates based on higher order shear deformation theory subjected to electromechanical loading. Navier's technique has been adopted for obtaining solutions of symmetric and anti-symmetric cross-ply and angle-ply laminates with simply supported boundary conditions. Newmark's method has been used for obtaining transient response of a laminated composite plate attached with piezoelectric layer. Effects of various parameters such as ply orientation, no of layers of composite laminated plates attached with piezoelectric layer subjected to electromechanical loadings are studied. Results predicted in this work are compared with those available in the literature.

Keywords: Smart materials, Navier's technique, Newmark's method.

Nomenclature

A	- Extension Stiffness Matrix.
B	- Bending-Extension Coupling Matrix.
D_S	- Elasticity matrix relating shear force and shear strains.
D_b	- Elasticity matrix relating moments and bending strains.
E_i	- Young's modulus of elasticity in the ith direction.
I	- Moment of inertia.
F_i	- In-plane force resultants.
M_r	- Moment resultants.
F_t	- Transverse force resultants.
σ	- Stress vector.
Q	- Elastic constant matrix.
ε	- Strain vector.
e	- Piezoelectric constant matrix.
E	- Electric field intensity vector.
Δt	-Time increment.
δU	- Virtual strain energy.
δV	- Virtual work done by applied forces.
δK	- Virtual kinetic energy.
z	- Distance of a point along the z-axis.
x, y, z	- Cartesian co-ordinates.
u, v, w	- Components of deformation in x, y, z axes.
$\varepsilon_o, \varepsilon_o^*$	- Strain components.
L, L^*	- Bending curvatures.
φ, φ *	- Transverse shear strains.

1. INTRODUCTION

S. J. Lee *et al* [1] has carried out transient analysis of composite laminated plates with embedded smart material layers using classical, first order and third order plate theories. They adopted numerical simulations using finite element method and also presented the results for studying the effects of boundary conditions, lamination scheme and loading. Y. Shindo *et al* [2] have analyzed scattering of time harmonic flexural waves in a symmetric piezoelectric laminated plate subjected to electric loading by applying dynamic theory of linear piezoelectricity. They used Fourier transforms for reducing problems to the solution of dual integral equations. Yoshihiro Ootao and Yoshinobu Tanigawa [3] has developed theoretical analysis of three dimensional transient piezothermoelasticity for rectangular composite plate composed of crystal class mm^2 and cross ply laminae subjected to partial heat supply. They obtained exact solutions for transient piezothermoelasticity of simply supported plate and temperature change in a transient state. Figures have been shown representing the numerical results stress, displacement, temperature change, electric displacement and electric potential distributions in a transient state. Yoshihiro Ootao *et al* [4] have developed a treatment for transient piezothermoelastic problem of cylindrical composite panel composed of crystal class mm^2 and angle ply laminae subjected to heat supply in circumferential direction. They have carried numerical calculations for angle ply composite laminated panel made of aluminum composite attached with piezoelectric layer of cadmium selenide solid. Chien-Ching Ma *et al* [5] have studied the transient analysis of piezoelectric bi-materials subjected to dynamic concentrated force and electric charge. Here problem is solved using Laplace transform and inverse Laplace transform method by means of Cagniard's method. They went for numerical calculations for examining transient behavior of field quantities in detail. B. L. Wang and N. Noda [6] have come to know the importance of piezoelectric materials also called as smart structures used in composite laminated plates to ensure structural rigidity and understanding the fracture of the structure. In order to reduce the problem to the solution of singular integral equations Fourier transformation technique has been used. They also investigated the influences of crack position on stress intensity factors and layer thickness. Ruifeng Wang *et al* [7] has developed 3 D finite element formulation for multiferroic composite and implemented into ABACUS software for its transient analysis. They showed that transient response can be influenced by input signal which could be tuned for strongest electric output. Jafar Rahiminasab and Jalil Rezaeepazhand [8] have analyzed transient vibration of 3 layer sandwich plate based on classical plate theory with electrorheological fluid (ER). They employed Hamilton's principle and used constant average acceleration scheme for deriving and integration of finite element equations of motion. From results they found that change in electric field will have influence on system natural frequencies. S. Pradyumna *et al* [9] employed higher order C^0 finite element formulation for analyzing transient analysis of functionally graded curved panels. They investigated transient analysis of functionally graded shell panels with varying of volume fraction index using simple power law distribution. Various studies have been carried on effects of different boundary conditions, geometry parameters and loadings. C. C. Hong [10] has investigated results of functionally graded material plate in thermal vibration and transient response using generalized differential quadrature (GDQ) method. He has obtained the real value solutions of functionally graded material plates with all edges are simply supported for center displacement. B. Biju *et al* [11] used 3D magnetic vector potential approach for presenting transient behavior of mild steel plate attached with magneto-electro-elastic sensors. In order to capture bending behavior of the plate, they used eight nodded brick element with number of elements across thickness direction. Thang Duy Vu and Ruediger Schmidt [12] have presented two nonlinear finite elements incorporated with piezoelectric layers based on first and third order shear deformation theory. For getting sensor output voltage of a piezolaminated plate several numerical tests are performed. Chen Zengtao *et al* [13] investigated transient response of two coplanar cracks in piezoelectric ceramic under in-plane electric and antiplane mechanical impacting loadings. Here electric displacement factors and dynamic stresses are obtained as functions of time and geometry parameters. From studies it is found that presence of electric field will enhance the crack propogation in piezoelectric ceramics at various stages of electromechanical load. J. N. Reddy *et al* [14] have developed computational tools for nonlinear analysis of smart composite systems with embedded piezoelectric layers. Their work mainly focuses on linear static analysis using FEM and nonlinear finite element formulation based on third order shear deformation theory. Effects of transverse normal stress/strains, transverse shear deformation has been incorporated by the theoretical model presented by Kant *et al* [15]. They adopted Navier's technique for obtaining solutions in closed form with solving boundary problem. Shiyekar *et al* [16] has presented analytical solutions for cross-ply composite laminates integrated with piezoelectric fiber-reinforced composite (PFRC) actuators which are under bidirectional bending. For analyzing smart material plates subjected to electromechanical loading, higher order shear and normal deformation theory (HOSNT12) has been used by them. Based on Hamilton's principle and finite element methods, linear response of piezothermoelastic plate has outlined by Fariborz Heidary *et al* [17]. They presented numerical results for a piezolaminated plate subjected to thermomechanical loadings. With use of electric potential difference across piezo layers, vibrations can be suppressed on piezolaminated composite plate.

2. FORMULATION

2.1. Displacement model

The higher order shear deformation theory with composite laminated plate attached with piezoelectric layer is [15]:

$$\left.\begin{aligned} u(x,y,z)&=u_o(x,y)+z\theta_x(x,y)+z^2u_o^*(x,y)+z^3\theta_x^*(x,y)\\ v(x,y,z)&=v_o(x,y)+z\theta_y(x,y)+z^2v_o^*(x,y)+z^3\theta_y^*(x,y)\\ w(x,y,z)&=w_o(x,y)\end{aligned}\right\} \quad (1)$$

Where

u_o, v_o and w_o denote the displacements of a point (x, y) on the midplane.

θ_x, θ_y are rotations of the normal to the midplane about y and x-axes.

u_0^*, v_0^*, θ_x^* and θ_y^* are representing the corresponding higher order deformation terms at the midplane.

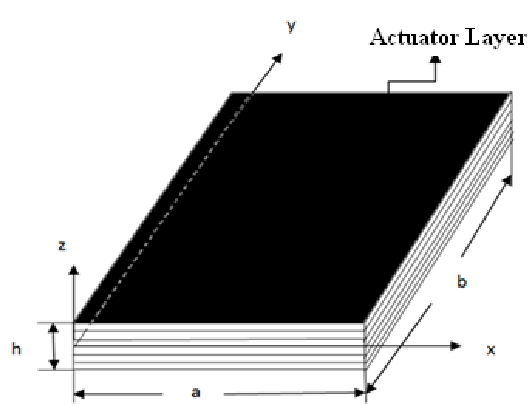

Figure-1. Composite laminated plate attached with piezoelectric layer [16]

The strain components are:

$$\begin{aligned} \varepsilon_x &= \varepsilon_{xo} + zk_x + z^2\varepsilon_{xo}^* + z^3k_x^*\\ \varepsilon_y &= \varepsilon_{yo} + zk_y + z^2\varepsilon_{yo}^* + 0z^3k_y^*\\ \varepsilon_z &= 0\end{aligned}$$

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$$\gamma_{xy} = \varepsilon_{xyo} + zk_{xy} + z^2\varepsilon^*_{xyo} + z^3 k^*_{xy}$$
$$\gamma_{yz} = \phi_y + z\varepsilon_{yzo} + z^2\phi^*_y$$
$$\gamma_{xz} = \phi_x + z\varepsilon_{xzo} + z^2\phi^*_x \tag{2}$$

The linear constitutive relations for elastic layer coupled with piezoelectric layer are [16]:

$$\{\sigma\} = [Q]\ \{\varepsilon\} - [e]\ \{E\} \tag{3}$$

The governing equations of displacement field are [17]:

$$\int_0^T (\delta U + \delta V - \delta K)\ dt = 0 \tag{4}$$

Substituting virtual strain energy (δU), virtual workdone (δV) and virtual kinetic energy (δK) in Equation (4) and integrating through the thickness of the laminate, and rewriting in-plane force, moment resultants, transverse force resultants and inertias in matrix, it is obtained as:

$$\begin{Bmatrix} F_i \\ F_i^* \\ -- \\ M_r \\ M_r^* \\ -- \\ F_t \\ F_t^* \end{Bmatrix} = \begin{bmatrix} \underline{A} & | & \underline{B} & | & \underline{0} \\ \underline{B^t} & | & \underline{D_b} & | & \underline{0} \\ 0 & | & 0 & | & D_s \end{bmatrix} \begin{Bmatrix} \varepsilon_o \\ \varepsilon_o^* \\ -- \\ L \\ L^* \\ -- \\ \phi \\ \phi^* \end{Bmatrix} \tag{5}$$

Following are the mechanical and electrical in-plane boundary conditions used for simply supported plate with both cross-ply (SS1) and angle-ply (SS2) laminates:

The SS1 boundary conditions for higher order shear deformation theory are

At edges x = 0 and x = a

$v_o = 0$, $w_o = 0$, $\theta_y = 0$, $M_x = 0$, $v_0^* = 0$, $\theta_y^* = 0$, $M_x^* = 0$, $N_x = 0$, $N_x^* = 0$, $\psi = 0$

At edges y = 0 and y = b

$u_o = 0$, $w_o = 0$, $\theta_x = 0$, $M_y = 0$, $u_0^* = 0$, $\theta_x^* = 0$, $M_y^* = 0$,

$N_y = 0$, $N_y^* = 0$, $\psi = 0$ (6)

The SS2 boundary conditions for higher order shear deformation theory are

At edges x = 0 and x = a

$u_0 = 0$, $w_o = 0$, $\theta_y = 0$, $N_{xy} = 0$, $M_x = 0$, $u_0^* = 0$, $\theta_y^* = 0$, $M_x^* = 0$, $N_{xy}^* = 0$, $\psi = 0$

At edges y = 0 and y = b

$v_0 = 0$, $w_o = 0$, $\theta_x = 0$, $N_{xy} = 0$, $M_y = 0$, $v_0^* = 0$, $\theta_x^* = 0$, $M_y^* = 0$, $N_{xy}^* = 0$, $\psi = 0$ (7)

In Newmark's integration method, the time derivatives are approximated, using difference approximations, and therefore solution is obtained only for discrete times and not as a continuous function of time.

$$\{\hat{K}\}\{u_{t+\Delta t}\} = \{\hat{F}_{es}\} + \{\hat{F}_{pz}\} = \{F\}$$

Where, $\{\hat{K}\} = [K] + a_3[M]$

$$\{F\} = [F]_{t+\Delta t} + [M](a_3\{u_t\} + a_4\{\dot{u}_t\} + a_5\{\ddot{u}\}) \tag{8}$$

The Equation (8) represents a system of algebraic equations among the discrete values of $\{u_t\}$ at time t to t+Δt in terms of known values at time. At the first time step t = 0, the values u_0, $\dot{u}_0, \ddot{u}_0$ are the initial known quantities. The transient response for both symmetric and anti-symmetric cross-ply and angle-ply laminates are estimated.

3. NUMERICAL RESULTS

A simply supported piezolaminated composite plate is considered for analysis. Composite plate is made of graphite/epoxy material. Piezoelectric material of PFRC attached at top of composite laminated plate.

Material properties of graphite/epoxy are [15]:

$$\frac{E_1}{E_2} = 25,\ \frac{G_{12}}{E_2} = 0.5,\ \frac{G_{23}}{E_2} = 0.2,\ E_2 = E_3 = 10^6\ N/cm^2$$
$$G_{12} = G_{13}\ and\ \mu_{12} = \mu_{23} = \mu_{13} = 0.25$$

Material properties for PFRC layer are [16]:

C_{11} = 32.6 GPa, $C_{12} = C_{21}$ = 4.3 GPa; $C_{13} = C_{31}$ = 4.76 GPa; $C_{22} = C_{33}$ = 7.2 GPa; C_{23} = 3.85 GPa; C_{44} = 1.05 GPa; $C_{55} = C_{66}$ = 1.29 GPa; e_{31} = - 6.76 C/m^2; $g_{11} = g_{22}$ = 0.037E - 9 C/V m; g_{33} = 10.64E - 9 C/V m.

Variation of displacement against time for simply supported symmetric and anti-symmetric cross-ply laminated plates attached with piezoelectric layer has been shown in Figure-2. The maximum percentage variation between present approach and S. J. Lee [1] is around 12%. Figure-3 demonstrates the effect of displacement with respect to time for symmetric and anti-symmetric angle-ply composite laminates attached with piezoelectric layer. In case of angle-ply laminates attached with piezo electric layer 10% is the maximum variation between present approach and S. J. Lee [1]. It is observed that the general angle-ply laminate shows less deflection, this may be because of it is attached with different ply orientations of laminae. Effect of position of piezo layer in anti-symmetric composite laminate has been demonstrated in Figure-4. Positioning of piezo layer at the top of composite laminate has great effect when compared to placing them in other positions. Various thicknesses of piezo layer and its effect have been clearly explained in Figure-5. Higher the thickness of the piezo layer, lesser the deflection because of reduction in vibration.

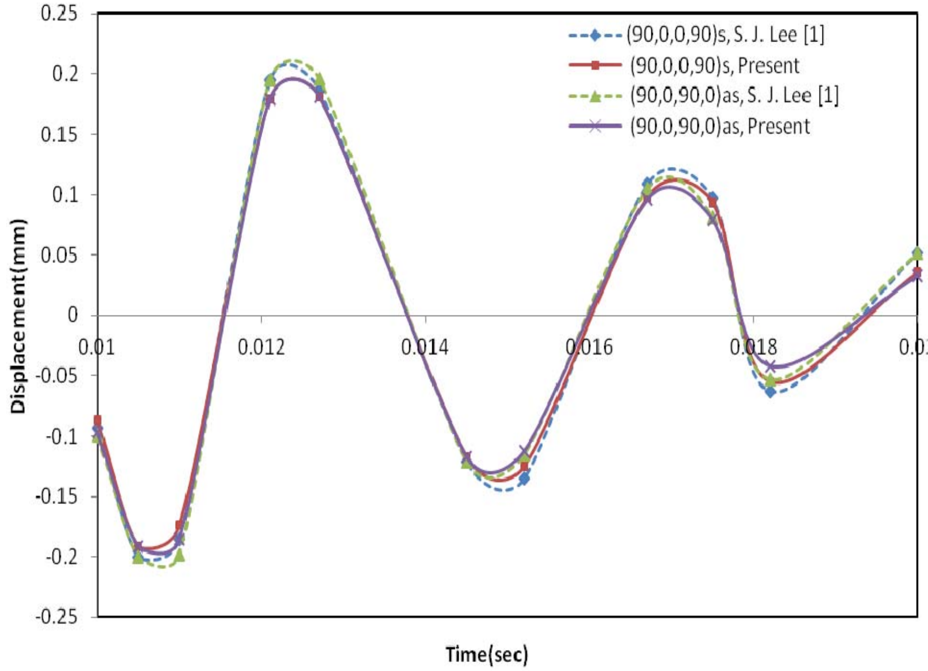

Figure-2. Effect of displacement against time for simply supported symmetric and anti-symmetric cross-ply laminated plates attached with piezo layer.

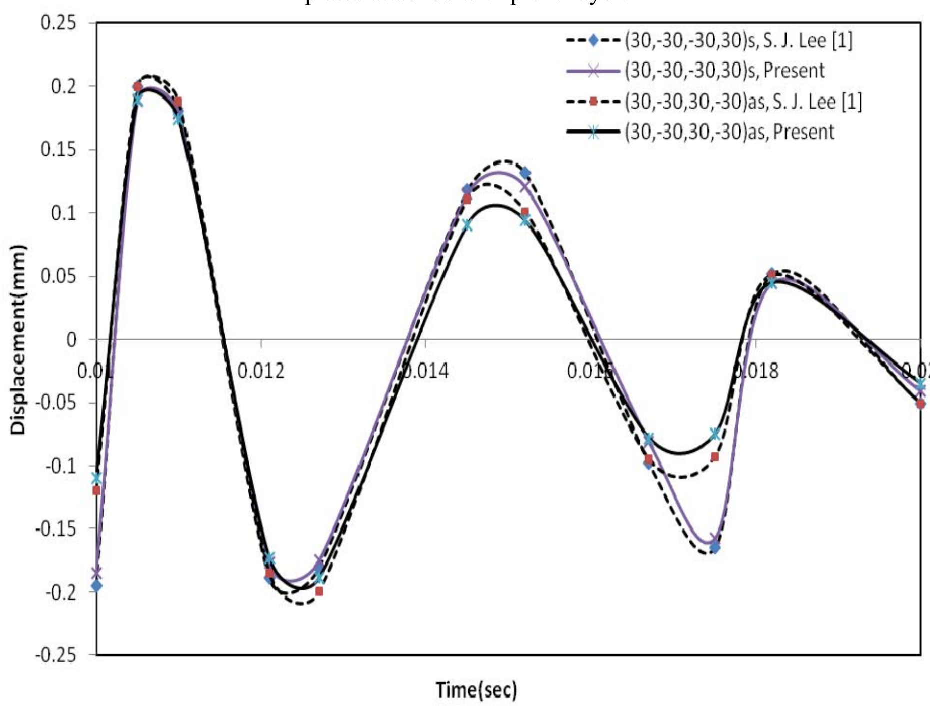

Figure-3. Effect of displacement against time for simply supported symmetric and anti-symmetric angle-ply laminated plates attached with piezo layer.

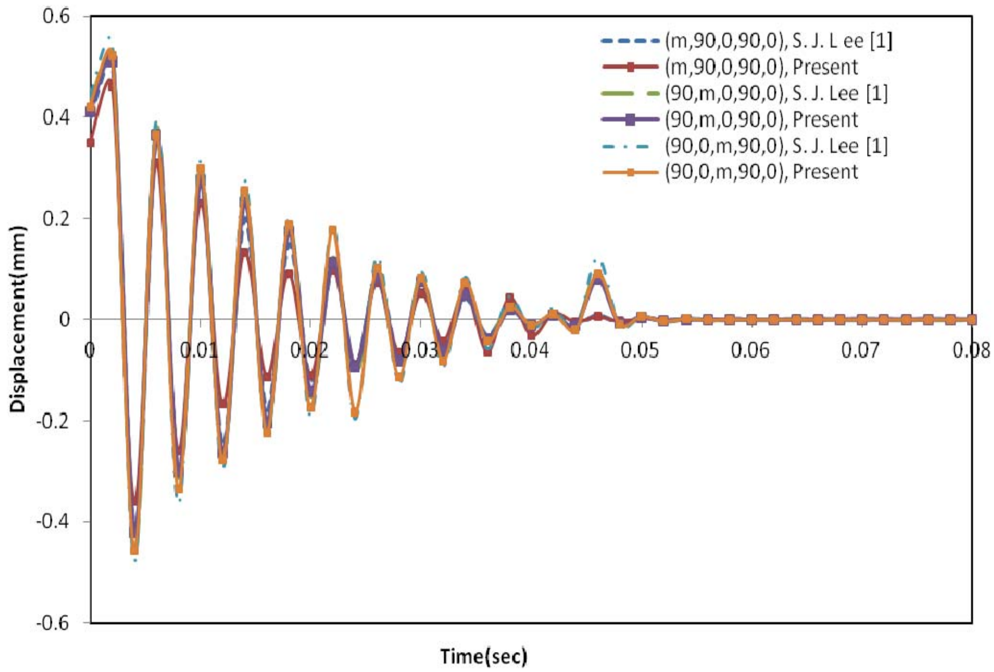

Figure-4. Effect of smart layer position in anti-symmetric composite laminated plate.

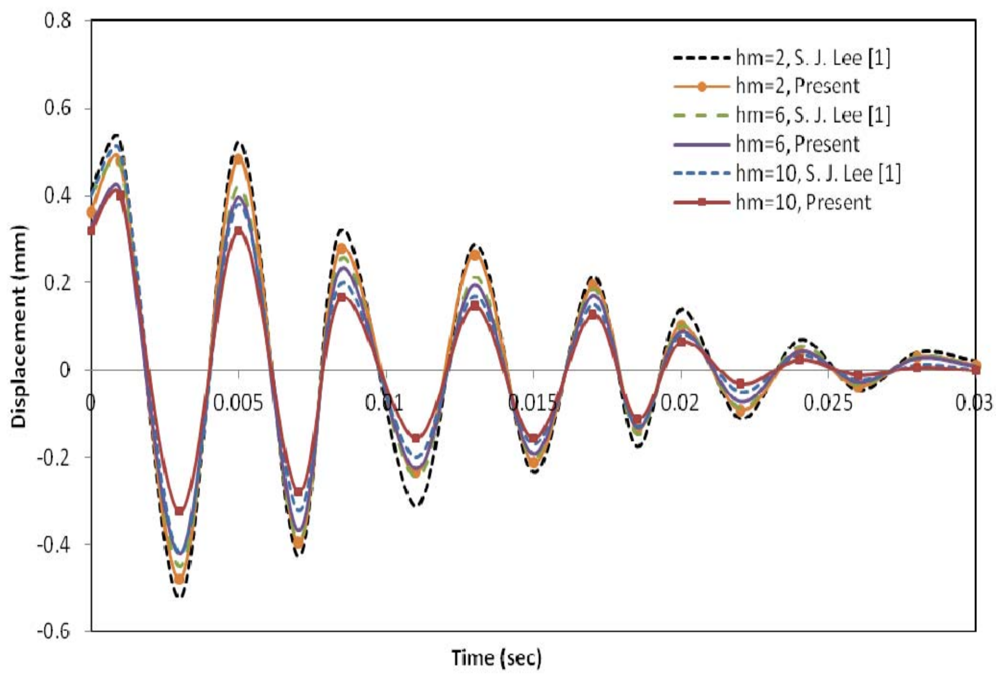

Figure-5. Effect of thickness of smart material layers on deflection damping characteristics of anti-symmetric cross-ply composite laminates.

4. CONCLUSIONS

In this paper an analytical procedure has been developed for piezolaminated composite plate subjected to electromechanical loading. Transient response of both symmetric and anti-symmetric cross-ply and angle-ply laminates attached with piezo layer has been analyzed. The results obtained for these laminates are in close agreement with available literature results. Various studies have been carried out on placing of piezo layer in anti-symmetric composite laminate and thickness variation of piezo layer in anti-symmetric cross-ply composite laminates.

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