ANALYTICAL METHOD OF OBTAINING INVERSE MODEL OF NON-LINEAR PROCESS USING FUZZY NONLINEAR INTERNAL MODEL CONTROL

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ABSTRACT

This novel proposes a modern approach for developing inverse model of a plant which will be based on fuzzy control logic. Purely analytical method was used in developing an inversion strategy of the plant. The proposed Fuzzy Nonlinear Internal Model Control (FNIMC) representation lies in the possibility to determine exactly the model inverse which grandees offset free control performances. The inverse model is conventionally having been performed using proportional-integral-derivative (PID) controller. This paper concludes by comparing the conventional method with Fuzzy Nonlinear Internal Model Control (FNIMC).

Keywords: fuzzy model, model inversion, proportional-integral-derivative controller, non-linear internal model control, fuzzy nonlinear internal model control.

1. INTRODUCTION

In the literature of Non-linear control strategies, several techniques have been developed and employed to control non-linearity in real time plants which are subjected to uncertainties and turbulences. One of the best method is Non-linear internal model control (NIMC) strategy which has played a significant role in industrial control applications due to its many desirable properties, in particular good robustness against disturbances and model mismatch\([1], [2], [3], [4], [7], [18]\).

The basic idea of non-linear internal model control (NIMC) introduced in \([2]\) is illustrated in Figure-1, where the controller can be directly obtained by model inversion. The robustness filter \(F_o\) is generally designed to alleviate sensitivity problems \([1], [5], [4], [6], [7]\).

In the implementation of NIMC, the determination of a plant model represents an important part of the development. For most complex practical applications, such as chemical and power plants, it represents a difficult task. Furthermore, the inversion of the non-linear model was shown to play a crucial role. Some authors studied analytical and numerical iterative algorithms for construction of non-linear operator inverses. \([2], [6]\). In this case, necessary and sufficient convergence conditions cannot be ensured. This problem is a great handicap in process control applications.

Developments of NIMC have been proposed for continuous- time systems \([8], [2]\) and for a class of discrete-time systems \([3], [6], [7]\). In this context, several NIMC schemes have been proposed. In \([2]\), an analytical inverse of a non-linear model, determined from a physical understanding of the plant, is proposed. In \([7]\), a neural network is trained to learn the inverse dynamics of the process and used as a non-linear controller. In \([9]\), the non-linear plant is represented by a collection of local linear models. Then, for each one the –optimal control design methodology is applied. Finally, the global controller is obtained by combining local linear controllers using a fuzzy system. In \([10]\), a fuzzy internal model controller for a class of linear with respect to the control input systems is proposed.

Generally, the developed NIMC methodologies impose at least one of the following restrictive conditions: the plant is linear with regard to the control input, the state of the plant is accessible for measurements \([17]\), the time interval between consecutive time instants is long enough to allow an iterative search for the control input, the plant has no zero dynamics, the plant operating region is limited to a sufficiently small neighborhood of the origin. The objective of this paper is to develop a NIMC method that is free of all these conditions. However, the proposed technique has to endure the following limitation the plant model must be invertible and have asymptotically stable zero dynamics \([11]\).

As control problems, which arise in a large variety of engineering fields, are characterized by uncertain environments and non-linearity, fuzzy systems seem to be a very effective tool to represent and control a wide class of complex non-linear systems whose complete mathematical models are not available. Recently, such fuzzy systems have been successfully used in solving non-linear control problems. Indeed, it is now recognized that fuzzy systems are capable to approximate any non-linear dynamics \([14], [16]\) on compact sets with an arbitrary degree of accuracy. The assumption that the plant representation exists is central and justifies the use of fuzzy models. It is the approach chosen in this paper where the process model is represented by a dynamical Takagi sugeno fuzzy system with constant conclusions (TSFSCC).

Fuzzy models usually used in control belong to the class of feed forward non-linear systems where the past measurements of the plant are taken as inputs of the fuzzy model \([2]\). In other words, when the plant output is fed into the fuzzy model, the corresponding system is basically a feed forward one. From a system theory point
of view, a feed forward fuzzy model represents a static non-linear mapping and its analysis requires the plant presence in Table-2. Another approach is to use dynamical models, included in a feedback configuration with time delays.

In this paper, the fuzzy model outputs are fed back as inputs to the model. Such a system represents a discrete-time dynamical system where the next-step estimates are functions of the previous ones only. This inversion strategy is developed based on purely analytical method. The proposed fuzzy non-linear internal model control (FNIMC) representation [13], [15] lies in the possibility to determine exactly the model inverse which grandees offset free control performances. The inverse process model to infer of immeasurable disturbances on shown in Figure-1. The IMC structure makes use of a discrete-time dynamical system where the next-step estimates are functions of the previous ones only. This mathematical representation of the real process and invertible, and then the controller is the inverse of the process model [12].

![Figure-1. Basic IMC structure.](image)

### 3. MATHEMATICAL MODEL OF PLANT AND CONTROLLER

\[
y(t) = \frac{g_p(s)g(s)}{1 + [g_p(s)-g(c)]g(s)}y(t) + \frac{1-g_p(s)g(c)}{1 + [g_p(s)-g(c)]g(s)}d(t)
\]

where
- \( g_p \) = Process transfer function
- \( g \) = Process model
- \( g_c \) = Controller
- \( g_d \) = Disturbances model.

The steady state gains are contained in [12] the definition of \( g \) itself. Ideally, the process model is a perfect representation of the real process and invertible, and then the condition is the inverse of the process model. The condition are given by

\[
g_c = g_p \text{ and } g_c^{-1} = g_p^{-1} \quad Y(s) = Y_r(s) \text{ for all } t > 0.
\]

That is, a perfect control could be obtained in the presence of any change in the set point and load disturbance.

### 4. PID CONTROLLER

The most common industrial controller is still the Proportional Integral Derivative (PID) controller. The basic equation of PID controller given by

\[
u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{d}{dt}e(t) \quad (3)
\]

where
- \( K_p \) = Proportional gain, a tuning parameter
- \( K_i \) = Integral gain, a tuning parameter
- \( K_d \) = Derivative gain, a tuning parameter
- \( e \) = Error
- \( t \) = Time or instantaneous time (the present)
- \( \Gamma \) = Variable of integration

There are significant advantages to not only designing the controller via IMC but also implementing it in the IMC structure as shown in Figure-1. The constraints in the manipulated variable can cause problem for the classical feedback structure. If the controller is implemented in the IMC structure, Input constraints (lower and upper limit for the input) will not create any problem if the actual (constrained) plant input is sent to the model rather than the actual value calculated by the controller. Then under the assumption that the model and the process transfer function are the same, the IMC structure remains open loop and stability guaranteed for the open loop stable process. The simple feedback structure may make the system unstable [11].
Figure-2. Nonlinear internal model control structure.

5. OVERVIEW

In this context, a possible alternative of Non-Linear Internal Model Control (NIMC) consists in developing a dynamical Takagi sugeno fuzzy system with constant conclusions (TSFSCC) of the plant from input–output data. The advantage of this approach is that the exact inverse model can be simply designed based on a fuzzy inversion.

In the fuzzy literature, many papers are concerned with fuzzy inversion. The first developed strategy consists in seeking the model inverse as a fuzzy system. This approach implements a principle of rule-by-rule inversion by permutation of the premise and consequence parts. Such a procedure allows obtaining an exact solution in the case of single-input–single-output (SISO) systems. However, its use for multiple-input–single-output (MISO) systems induces incomplete rule bases whose filling requires the definition of fictitious symbols whose membership function supports come out of the fixed universe of discourse. Indeed, it can be shown that the analytical inversion of TSFSCC leads to a homographic function which cannot be exactly represented by a TSFSCC.

A second strategy for the inversion consists in using an iterative method to carry out the inverse model. Various techniques can then be implemented (binary search descent of gradient, combination of genetic algorithms and Newton method). The disadvantage of this type of approach is that the provided solution is not exact and the algorithm convergence cannot be guaranteed. Moreover, the iterative nature induces a significant computation time which must be repeated for each new input vector. Lastly, whatever the methodology chosen, the case of multiple solutions of inversion is ignored.

Direct inversion is generally restricted to monotonous rule bases for which the solution utility is of multiple solutions results in a convergence to a particular solution which differs according to imposed initial conditions. In this Project, a methodology is thus proposed to dynamically invert the fuzzy model. In order to deal with several acceptable solutions, the inversion is approached in a local way, i.e., on the elementary subsystems capable of providing a solution. By doing so, the inversion of the global fuzzy system is tackled by inverting some of its components. It is important to note here that the developed inversion strategy is exact and based on a purely analytical methodology. It thus deviates from both approaches previously discussed.

Finally, a control methodology based on a fuzzy non-linear internal model control (FNIMC) architecture is proposed. The main advantage of the chosen fuzzy representation lies in the possibility to determine exactly the model inverse which guarantees offset-free control performances. It should however be noted that the developed method is still restricted to SISO processes with stable inverses.

Figure-3. Fuzzy nonlinear internal model controller.

A. Fuzzy control with inverse dynamics

In the linear control context, when an inverse dynamics control strategy is used, the resulting control law solution is unique and ensures the system invisibility and the stability of the control structure if and only if the system is minimum phase (the zeros of the system are strictly inside the unit circle). In this case, no control choice criterion is needed. In the non-linear case, several control law solutions produced by the inversion can co-exist. In this case, the choice criterion influence on the control synthesis must be studied.

It is assumed that a sequence of inputs Uo (K) can be chosen so that the model output is equal to zero for all. In this case, for a given fuzzy system in the form the zero dynamics behavior can be characterized by the sequence which is chosen so that is equal to zero. In other words, according to the sequence satisfies the following difference non-linear equation:

$$u_o(k) - \Psi_1(\lambda)[0,...,0,u_o(k-1),...,u_o(k-f+1)]$$

$$\Psi_2(\lambda)[0,...,0,u_o(k-1),...,u_o(k-f+1)]$$

B. Fuzzy system without zero dynamics

If the model has non zero dynamics, the feedback control inputs are not used by the fuzzy inverse. Indeed, only and are fed into the fuzzy inverse. Therefore, [3] becomes [11]. In this case, if the desired input is admissible i.e., it belongs to the output universe of discourse, and no model singularities i.e., are present, the inversion problem has always one or several solutions. In case of multiple solutions, a criterion has to be considered. Of course, it
will influence the control signal behavior, however it cannot make the model non invisible.

![Figure-4. Membership function of temperature function.](image1)

Criteria leading to high frequency control inputs (abrupt switching between fuzzy meshes) or to discontinuous ones should be avoided because such control laws can damage actuators. In this the minimal energy criterion is used because it is relevant to the physical properties of the actuators. Therefore, the selected control input at sampling-time is the nearest to that at sampling-time, thus avoiding abrupt switching and ensuring control law continuity.

\[
0 \leq u_{0}(k) = \frac{\psi_{1}(\lambda)[0, \ldots, 0]}{\psi_{2}(\lambda)[0, \ldots, 0]}
\]  

(8)

C. Fuzzy system with zero dynamics

Indeed, the fuzzy inverse is used in feedback control input configurations. In other words, the past values of are integrated in the fuzzy inversion synthesis and the zero dynamics is given by [3].

![Figure-5. Membership function of change in error.](image2)

In this case, unlike to the previous one, the inversion problem has not always a solution. Since, the past values of are fed back to the model inverse, the criterion now has an important influence on the inversion problem.

Let be the set of possible solutions of [7] at time. Let the mapping associated with the choice criterion that selects one value in the set. Here, it clearly appears that the input vector in [3] relies on the past choices induced by the criterion that is Depending on the choice criterion, abrupt switching between meshes can lead to an unstable model inverse. Indeed, input may be outside the control universe of discourse. In this case, the control action does not belong to any fuzzy elementary subsystem and the control problem has no solution. Actually, there is no general approach for selecting the criterion. So, the effect of a given criterion will be experimentally characterized by observing the behavior.

![Figure-6. Output membership function.](image3)

The solution of the non-linear equation converges asymptotically to zero, the fuzzy model has asymptotically stable zero dynamics. However, if the solution of this equation is unstable, the zero dynamics are un-stable and the fuzzy model is not invertible.

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6. ROBUSTNESS OF THE STABILITY

It can be noted that the filter parameter adjustment allows tuning stability robustness. Indeed, using the FNIMC structure Illustrated in Figure-2, the process output is given by (2) the un-modeled dynamics effect on stability at high frequencies Contained in the term (4) can be attenuated by decreasing the filter bandwidth. By comparison between linear and non-linear cases, it can be thus concluded that the filter plays the same role as the robustness filter in the linear internal model control, which guarantees the robustness of stability with respect to plant-model mismatch.

7. RESULT AND DISCUSSIONS

The response of a PID Controller and FPC Control for ideal condition shows the response of an increased time constant.
The Primary Loop shows the Performance of PID Controller and FNIMC Controller (Simulation) by comparing the above results it can be concluded that the response obtained using FNIMC is better than the conventional PID controller. Even if the process parameter changes the FNIMC adapts itself and produces good response. Hence it acts as an adaptive controller. Figure 7 shows the response of FNIMC and PID controller under continuous load disturbances. It is observed that conventional PID controller is not able to follow the set point but FNIMC controller follows the set point. The conventional PID controller gives a damped oscillation for non-relevant tuning parameter with continuous load disturbances but FNIMC produces good response. Hence FNIMC does not require precise tuning.

CONCLUSIONS

It is shown that the FNIMC controller can be built from the exact fuzzy model inverse connected in series with a robustness filter. The filter objective is to provide a compromise between performance and robustness of the control structure. The inversion technique, presented for a class of fuzzy systems, consists in decomposing the fuzzy system into elementary subsystems. Then, the inversion problem is solved in a local way by inverting all subsystems able to provide an inverse solution. The proposed control strategy ensures offset-free Performances. The advantage of the FNIMC is its robustness with respect to model mismatches and disturbances.

The design methodology was illustrated on simulated plants and can potentially be applied to a wide class of process control problems. The developed FNIMC strategy is restricted to systems with stable inverses (systems with asymptotically stable zero dynamics). However, FNIMC can be applied to processes with unstable inverses if a minimum phase model is combined with a pure time-delay.

Future work will focus on the extension of this approach to control processes with unstable inverses by using non-linear model predictive control techniques.

REFERENCES


